AP® Calculus BC
Practice Exam

From the 2016 Administration

Contents

Note: This publication shows the page numbers that appeared in the *2015–16 AP Exam Instructions* book and in the actual exam. This publication was not repaginated to begin with page 1.

Exam Instructions

The following contains instructions taken from the *2015–16 AP Exam Instructions* book.

AP® Calculus AB/BC Exam

Regularly Scheduled Exam Date: Thursday morning, May 5, 2016
Late-Testing Exam Date: Thursday morning, May 19, 2016

Section I Total Time, Calculus AB: 1 hr. 45 min. Section II Total Time, Calculus AB: 1 hr. 30 min.
Section I Total Time, Calculus BC: 1 hr. 45 min. Section II Total Time, Calculus BC: 1 hr. 30 min.

Section I

Total Time: 1 hour 45 minutes
Number of Questions: 45*
Percent of Total Score: 50%
Writing Instrument: Pencil required

*The number of questions may vary slightly depending on the form of the exam.

Part A:
Number of Questions: 28
Time: 55 minutes
No calculator allowed

Part B:
Number of Questions: 17
Time: 50 minutes
Graphing calculator required

Section II

Total Time: 1 hour 30 minutes
Number of Questions: 6
Percent of Total Score: 50%
Writing Instrument: Either pencil or pen with black or dark blue ink

Note: For Section II, if students finish Part A before the end of the timed 30 minutes for Part A, they cannot begin working on Part B. Students must wait until the beginning of the timed 60 minutes for Part B. However, during the timed portion for Part B, students may work on the problems in Part A without the use of a calculator.

Part A:
Number of Questions: 2
Time: 30 minutes
Percent of Section II Score: 33.3%
Graphing calculator required

Part B:
Number of Questions: 4
Time: 60 minutes
Percent of Section II Score: 66.6%
No calculator allowed

What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- *2015-16 AP Coordinator's Manual*
- This book — *AP Exam Instructions*
- AP Exam Seating Chart template(s)
- School Code and Home-School/Self-Study Codes
- Extra graphing calculators
- Pencil sharpener
- Container for students' electronic devices (if needed)
- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
  – "Exam in Progress"
  – "Cell phones are prohibited in the testing room"

SEATING POLICY FOR AP CALCULUS AB AND CALCULUS BC EXAMS

Testing Window	Exams Administered at Schools in the United States, Canada, Puerto Rico, and the U.S. Virgin Islands	Exams Administered at Schools Outside the United States, Canada, Puerto Rico, and the U.S. Virgin Islands
Regularly Scheduled Exams	**Students must be seated no less than four feet apart.**	**Students must be seated no less than five feet apart.**
Late-Testing Exams	**Students must be seated no less than five feet apart.**	

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 47 of the *2015-16 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 45 of the *AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

■ **Do not begin the exam instructions below until you have completed the appropriate**
● **General Instructions for your group.**

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time. Remember, you must complete a seating chart for this exam. See pages 305–306 for a seating chart template and instructions. See the *2015-16 AP Coordinator's Manual* for exam seating requirements (pages 49–52).

If you are giving the regularly scheduled exam, say:

> **It is Thursday morning, May 5, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.**

If you are giving the alternate exam for late testing, say:

> **It is Thursday morning, May 19, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.**

> **In a moment, you will open the packet that contains your exam materials.**

> **By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2015-16 Bulletin for AP Students and Parents*. Please check to make sure you have the correct exam: Calculus AB or Calculus BC. Raise your hand if you do not have the correct exam. . . .**

> **You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .**

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> **Carefully remove the AP Exam label found near the top left of your exam booklet cover. Now place it on page 1 of your answer sheet on the light blue box near the top right-hand corner that reads "AP Exam Label."**

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam can still be processed correctly.

> **Read the statements on the front cover of Section I and look up when you have finished. . . .**

> **Sign your name and write today's date. Look up when you have finished. . . .**

> **Now print your full legal name where indicated. Are there any questions? . . .**

> **Turn to the back cover of your exam booklet and read it completely. Look up when you have finished. . . .**

> **Are there any questions? . . .**

> **You will now take the multiple-choice portion of the exam. You should have in front of you the multiple-choice booklet and your answer sheet. Open your answer sheet to page 2. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled.**

> **You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.**

> **Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .**

> **You have 55 minutes for Part A. Part A questions are numbered 1 through 28. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.**

🕐 Note Start Time here _____. Note Stop Time here _____. Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 45 minutes, say:

> **There are 10 minutes remaining.**

After 10 minutes, say:

> **Stop working on Part A and turn to page 22 in your Section I booklet. . . .**

> **On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out**

of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 92. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 50 minutes for Part B. You may begin.

Note Start Time here _____. Note Stop Time here _____. Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 40 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working and turn to page 38. You have 3 minutes to answer Questions 93–96. These are survey questions and will not affect your score. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions. Then say:

Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. After all answer sheets have been collected, say:

Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, notes, or textbooks during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any

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electronic or communication device. Remember, you may never discuss the multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled. Are there any questions? . . .

🕐 **You may begin your break. Testing will resume at _____.**

SECTION II: Free Response

After the break, say:

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using a pen with black or dark blue ink, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and, using your pen, complete Item 1 under "Important Identification Information." Print the first two letters of your <u>last</u> name and the first letter of your <u>first</u> name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

I need to collect the Student Pack from anyone who will be taking another AP Exam. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

Read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

> **Are there any questions? . . .**
>
> **Section II also has two parts that are timed separately. You are responsible for pacing yourself and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions? . . .**
>
> **You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra sheet of paper you use, be sure to write only your AP number and the question number you are working on. Do not write your name. Open your exam booklet and begin.**

🕐 Note Start Time here ——————. Note Stop Time here ——————. Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1s or 2s at the top of each page to assist you in monitoring their work. After 20 minutes, say:

> **There are 10 minutes remaining in Part A.**

After 10 minutes, say:

> **Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .**
>
> **Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may <u>not</u> use your calculator. Remember to show your work and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .**
>
> **Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may begin Part B.**

🕐 Note Start Time here ——————. Note Stop Time here ——————. After 50 minutes, say:

> **There are 10 minutes remaining in Part B.**

After 10 minutes, say:

> **Stop working and close your exam booklet. Place it on your desk, face up. . . .**

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their exam booklets. Complete an Incident Report. A single Incident Report may be completed for multiple students per exam subject per administration (regular or late testing) as long as all of the required information is provided. Include all exam booklets with extra sheets of paper in an Incident Report return envelope (see page 60 of the *2015-16 AP Coordinator's Manual* for complete details). Then say:

> **Remain in your seat, without talking, while the exam materials are collected. . . .**

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Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed his or her initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

You may not discuss or share these specific free-response questions with anyone unless they are released on the College Board website in about two days. Your AP Exam score results will be available online in July.

If you are giving the alternate exam for late testing, say:

None of the questions in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

All exam materials must be placed in secure storage until they are returned to the AP Program after your school's last administration. Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:

- Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See "Post-Exam Activities" in the *2015-16 AP Coordinator's Manual.*
- Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Be sure to give the completed seating chart to the AP Coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

Student Answer Sheet for the Multiple-Choice Section

Use this section to capture student responses. (Note that the following answer sheet is a sample, and may differ from one used in an actual exam.)

♥ **CollegeBoard**

AP Exam Label
(from Section I Booklet)

AP®

Answer Sheet 2016

AP Number Label
(from Student Pack)

B1234567789T

COMPLETE THIS AREA AT EVERY EXAM.

USE NO. 2 PENCIL ONLY

To maintain the security of the exam and the validity of my AP score, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after completing the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the *2015-16 Bulletin for AP Students and Parents*, including using testing accommodations (e.g., extended time, computer, etc.) only if I have been preapproved by College Board Services for Students with Disabilities.

A. SIGNATURE Sign your legal name as it will appear on your college applications.   Date

B. LEGAL NAME   Omit apostrophes, Jr., II.

Legal Last Name — First 15 Letters

Legal First Name — First 12 Letters   MI

Exam Name:

H. AP EXAM I AM TAKING USING THIS ANSWER SHEET

Form: Form Code:

785208

SCHOOL USE ONLY

Section Number   ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Fee Reduction Granted   ① Option 1   ② Option 2

C. YOUR AP NUMBER

D. EXAM DATE   Month   Day

E. EXAM START TIME   AM   PM

F. MULTIPLE-CHOICE BOOKLET SERIAL NUMBER   S

COMPLETE THIS AREA ONLY ONCE.

I. AREA CODE AND PHONE NUMBER

INTERNATIONAL PHONE

J. SCHOOL YOU ATTEND   SCHOOL CODE   School Name

City

State

Country

K. DATE OF BIRTH   Month   Day   Year

Jan   Feb   Mar   Apr   May   Jun   Jul   Aug   Sep   Oct   Nov   Dec

L. SOCIAL SECURITY NUMBER (Optional)

M. COLLEGE TO RECEIVE YOUR AP SCORE REPORT   COLLEGE CODE   Using the college code listed in the AP Student Pack, indicate the ONE college that you want to receive your AP score report.

College Name

City

State

Country

N. CURRENT GRADE LEVEL

○ Not yet in 9th grade
○ 9th
○ 10th
○ 11th
○ 12th
○ No longer in high school

O. STUDENT SEARCH SERVICE®

Colleges and scholarship programs may request your information to inform you of educational opportunities and financial aid.

Would you like us to supply your information?

○ Yes   ○ No

If you don't answer and previously chose to participate in this service, we will continue providing your information.

COMPLETE THIS AREA AT EACH EXAM (IF APPLICABLE).

P. SURVEY QUESTIONS — Answer the survey questions in the AP Student Pack. Do not put responses to exam questions in this section.

1. (A) (B) (C) (D) (E) (F) (G) (H) (I)
2. (A) (B) (C) (D) (E) (F) (G) (H) (I)
3. (A) (B) (C) (D) (E) (F) (G) (H) (I)

4. (A) (B) (C) (D) (E) (F) (G) (H) (I)
5. (A) (B) (C) (D) (E) (F) (G) (H) (I)
6. (A) (B) (C) (D) (E) (F) (G) (H) (I)

7. (A) (B) (C) (D) (E) (F) (G) (H) (I)
8. (A) (B) (C) (D) (E) (F) (G) (H) (I)
9. (A) (B) (C) (D) (E) (F) (G) (H) (I)

Q. LANGUAGE — Do not complete this section unless instructed to do so.

If this answer sheet is for the French Language and Culture, German Language and Culture, Italian Language and Culture, Spanish Language and Culture, or Spanish Literature and Culture Exam, please answer the following questions. Your responses will not affect your score.

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?

○ Yes   ○ No

2. Do you regularly speak or hear the language at home?

○ Yes   ○ No

QUESTIONS 1–75

Indicate your answers to the exam questions in this section (pages 2 and 3). Mark only one response per question for Questions 1 through 120. If a question has only four answer options, do not mark option E. Answers written in the multiple-choice booklet will not be scored.

COMPLETE MARK ●   **EXAMPLES OF INCOMPLETE MARKS**

You must use a No. 2 pencil and marks must be complete. Do not use a mechanical pencil. It is very important that you fill in the entire circle darkly and completely. If you change your response, erase as completely as possible. Incomplete marks or erasures may affect your score.

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)
16. (A) (B) (C) (D) (E)
17. (A) (B) (C) (D) (E)
18. (A) (B) (C) (D) (E)
19. (A) (B) (C) (D) (E)
20. (A) (B) (C) (D) (E)
21. (A) (B) (C) (D) (E)
22. (A) (B) (C) (D) (E)
23. (A) (B) (C) (D) (E)
24. (A) (B) (C) (D) (E)
25. (A) (B) (C) (D) (E)

26. (A) (B) (C) (D) (E)
27. (A) (B) (C) (D) (E)
28. (A) (B) (C) (D) (E)
29. (A) (B) (C) (D) (E)
30. (A) (B) (C) (D) (E)
31. (A) (B) (C) (D) (E)
32. (A) (B) (C) (D) (E)
33. (A) (B) (C) (D) (E)
34. (A) (B) (C) (D) (E)
35. (A) (B) (C) (D) (E)
36. (A) (B) (C) (D) (E)
37. (A) (B) (C) (D) (E)
38. (A) (B) (C) (D) (E)
39. (A) (B) (C) (D) (E)
40. (A) (B) (C) (D) (E)
41. (A) (B) (C) (D) (E)
42. (A) (B) (C) (D) (E)
43. (A) (B) (C) (D) (E)
44. (A) (B) (C) (D) (E)
45. (A) (B) (C) (D) (E)
46. (A) (B) (C) (D) (E)
47. (A) (B) (C) (D) (E)
48. (A) (B) (C) (D) (E)
49. (A) (B) (C) (D) (E)
50. (A) (B) (C) (D) (E)

51. (A) (B) (C) (D) (E)
52. (A) (B) (C) (D) (E)
53. (A) (B) (C) (D) (E)
54. (A) (B) (C) (D) (E)
55. (A) (B) (C) (D) (E)
56. (A) (B) (C) (D) (E)
57. (A) (B) (C) (D) (E)
58. (A) (B) (C) (D) (E)
59. (A) (B) (C) (D) (E)
60. (A) (B) (C) (D) (E)
61. (A) (B) (C) (D) (E)
62. (A) (B) (C) (D) (E)
63. (A) (B) (C) (D) (E)
64. (A) (B) (C) (D) (E)
65. (A) (B) (C) (D) (E)
66. (A) (B) (C) (D) (E)
67. (A) (B) (C) (D) (E)
68. (A) (B) (C) (D) (E)
69. (A) (B) (C) (D) (E)
70. (A) (B) (C) (D) (E)
71. (A) (B) (C) (D) (E)
72. (A) (B) (C) (D) (E)
73. (A) (B) (C) (D) (E)
74. (A) (B) (C) (D) (E)
75. (A) (B) (C) (D) (E)

ETS USE ONLY

Exam: (0)(1)(2)(3)(4)(5)(6)(7)(8)(9) (0)(1)(2)(3)(4)(5)(6)(7)(8)(9)
Exam: (0)(1)(2)(3)(4)(5)(6)(7)(8)(9) (0)(1)(2)(3)(4)(5)(6)(7)(8)(9)

SELECTED MEDIA EXAMS	R	W	O	OTHER EXAMS	R	W	O
PT02				TOTAL			
PT03				Subscore (if applicable)			
PT04				Subscore (if applicable)			

DO NOT WRITE IN THIS AREA

QUESTIONS 76–120

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

76 (A)(B)(C)(D)(E)	91 (A)(B)(C)(D)(E)	106 (A)(B)(C)(D)(E)
77 (A)(B)(C)(D)(E)	92 (A)(B)(C)(D)(E)	107 (A)(B)(C)(D)(E)
78 (A)(B)(C)(D)(E)	93 (A)(B)(C)(D)(E)	108 (A)(B)(C)(D)(E)
79 (A)(B)(C)(D)(E)	94 (A)(B)(C)(D)(E)	109 (A)(B)(C)(D)(E)
80 (A)(B)(C)(D)(E)	95 (A)(B)(C)(D)(E)	110 (A)(B)(C)(D)(E)
81 (A)(B)(C)(D)(E)	96 (A)(B)(C)(D)(E)	111 (A)(B)(C)(D)(E)
82 (A)(B)(C)(D)(E)	97 (A)(B)(C)(D)(E)	112 (A)(B)(C)(D)(E)
83 (A)(B)(C)(D)(E)	98 (A)(B)(C)(D)(E)	113 (A)(B)(C)(D)(E)
84 (A)(B)(C)(D)(E)	99 (A)(B)(C)(D)(E)	114 (A)(B)(C)(D)(E)
85 (A)(B)(C)(D)(E)	100 (A)(B)(C)(D)(E)	115 (A)(B)(C)(D)(E)
86 (A)(B)(C)(D)(E)	101 (A)(B)(C)(D)(E)	116 (A)(B)(C)(D)(E)
87 (A)(B)(C)(D)(E)	102 (A)(B)(C)(D)(E)	117 (A)(B)(C)(D)(E)
88 (A)(B)(C)(D)(E)	103 (A)(B)(C)(D)(E)	118 (A)(B)(C)(D)(E)
89 (A)(B)(C)(D)(E)	104 (A)(B)(C)(D)(E)	119 (A)(B)(C)(D)(E)
90 (A)(B)(C)(D)(E)	105 (A)(B)(C)(D)(E)	120 (A)(B)(C)(D)(E)

QUESTIONS 121–126

For Students Taking AP Biology

Write your answer in the boxes at the top of the griddable area and fill in the corresponding circles.
Mark only one circle in any column. You will receive credit only if the circles are filled in correctly.

121   122   123   124   125   126

(Griddable answer grids with digits 0–9, decimal points, slashes, and negative signs)

QUESTIONS 131–142

For Students Taking AP Physics 1 or AP Physics 2

Mark two responses per question. You will receive credit only if both correct responses are selected.

131 (A)(B)(C)(D)	135 (A)(B)(C)(D)	139 (A)(B)(C)(D)
132 (A)(B)(C)(D)	136 (A)(B)(C)(D)	140 (A)(B)(C)(D)
133 (A)(B)(C)(D)	137 (A)(B)(C)(D)	141 (A)(B)(C)(D)
134 (A)(B)(C)(D)	138 (A)(B)(C)(D)	142 (A)(B)(C)(D)

DO NOT WRITE IN THIS AREA

COMPLETE THIS AREA ONLY ONCE.

R. YOUR MAILING ADDRESS
Use the address abbreviations from your AP Student Pack. Fill in only one circle per column. Indicate a space in your address by leaving a blank box; do not grid that column.

STREET ADDRESS (include street number, street name, apartment number, etc.)

CITY

ZIP OR POSTAL CODE

COUNTRY CODE

STATE

AK AL AR AZ CA CO CT DC DE FL GA HI IA ID IL IN KS KY LA MA MD ME MI MN MO MS MT NC ND NE NH NJ NM NV NY OH OK OR PA RI SC SD TN TX UT VA VT WA WI WV WY Puerto Rico AA AE AP Other

V. SEX
○ Female ○ Male

W. WHICH LANGUAGE DO YOU KNOW BEST?
○ English
○ English and another language about the same
○ Another language

X. RACIAL/ETHNIC GROUP
Please answer both questions about Hispanic origin and about race. For the following questions about your identity, Hispanic origins are not races.

(You may mark all that apply.)

a. Are you of Hispanic, Latino, or Spanish origin?
○ No, not of Hispanic, Latino, or Spanish origin
○ Yes, Cuban
○ Yes, Mexican
○ Yes, Puerto Rican
○ Yes, another Hispanic, Latino, or Spanish origin

b. What is your race?
○ American Indian or Alaska Native
○ Asian (including Indian subcontinent and Philippines origin)
○ Black or African American (including Africa and Afro-Caribbean origin)
○ Native Hawaiian or other Pacific Islander
○ White (including Middle Eastern origin)

Y. PARENTAL EDUCATION LEVEL
In the first column, indicate the highest level of education of one parent/guardian, and indicate whether this is your mother/female guardian or father/male guardian. Then, if applicable, indicate the highest level of education of your other parent/guardian in the second column, and indicate whether this is your mother/female guardian or father/male guardian.

○ Mother or female guardian
○ Father or male guardian

○ Grade school
○ Some high school
○ High school diploma or equivalent
○ Vocational or trade school
○ Some college
○ Associate or two-year degree
○ Bachelor's or four-year degree
○ Some graduate or professional school
○ Graduate or professional degree

T. STUDENT IDENTIFIER (Student ID Number)

S. FOR STUDENTS OUTSIDE THE UNITED STATES ONLY
○ If your address does not fit in the spaces provided in Item R, fill in as many circles as you can, then fill in the circle in Item S and print the remainder of your address in the spaces provided.

Address   City   State or Province   ZIP or Postal Code   Country

U. EMAIL ADDRESS

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2016 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION I: Multiple Choice

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question   Sample Answer

Chicago is a   Ⓐ ● Ⓒ Ⓓ Ⓔ
(A) state
(B) city
(C) country
(D) continent
(E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4LBP6-S

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\sin(\pi x)}$ is

(A) -2 (B) $-\dfrac{2}{\pi}$ (C) 0   (D) $\dfrac{2}{\pi}$ (E) nonexistent

2. For time $t > 0,$ the position of a particle moving in the xy-plane is given by the vector $\left\langle \dfrac{1}{t}, e^{3t} \right\rangle.$ What is the velocity vector of the particle at time $t = 2$?

(A) $\left\langle \dfrac{1}{2}, e^6 \right\rangle$ (B) $\left\langle \dfrac{1}{4}, e^6 \right\rangle$ (C) $\left\langle \dfrac{1}{4}, 3e^6 \right\rangle$ (D) $\left\langle -\dfrac{1}{4}, e^6 \right\rangle$ (E) $\left\langle -\dfrac{1}{4}, 3e^6 \right\rangle$

GO ON TO THE NEXT PAGE.

3. Consider the series $\sum_{n=1}^{\infty} a_n$. If $a_1 = 16$ and $\frac{a_{n+1}}{a_n} = \frac{1}{2}$ for all integers $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is

   (A) 0   (B) 2   (C) 17   (D) 32   (E) divergent

4. If $y = \left(\frac{x}{x+1}\right)^5$, then $\frac{dy}{dx} =$

   (A) $5(1+x)^4$ (B) $\frac{x^4}{(x+1)^4}$ (C) $\frac{5x^4}{(x+1)^4}$ (D) $\frac{5x^4}{(x+1)^6}$ (E) $\frac{5x^4(2x+1)}{(x+1)^6}$

GO ON TO THE NEXT PAGE.

5. Which of the following gives the total area enclosed by the graph of the polar curve $r = \theta \sin 2\theta$ for $0 \le \theta \le 2\pi$?

(A) $\int_0^{2\pi} \frac{1}{2} |\theta \sin 2\theta| \, d\theta$

(B) $\int_0^{2\pi} |\theta \sin 2\theta| \, d\theta$

(C) $\int_0^{2\pi} \frac{1}{2} (\theta \sin 2\theta)^2 \, d\theta$

(D) $\int_0^{2\pi} (\theta \sin 2\theta)^2 \, d\theta$

(E) $\int_0^{2\pi} \frac{\pi}{2} (\theta \sin 2\theta)^2 \, d\theta$

6. The slope of the line tangent to the graph of $y = \ln(1 - x)$ at $x = -1$ is

(A) -1 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) $\ln 2$ (E) 1

GO ON TO THE NEXT PAGE.

x	2	2.2	2.4
$f'(x)$	−0.5	−0.3	−0.1

7. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = f'(x)$ with initial condition $f(2) = 3$. Selected values of f' are given in the table above. What is the approximation for $f(2.4)$ if Euler's method is used, starting at $x = 2$ with two steps of equal size?

   (A) 2.80   (B) 2.82   (C) 2.84   (D) 2.92   (E) 3.16

8. $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2 + 9}}\,dx =$

   (A) −2   (B) $-\dfrac{2}{15}$ (C) 1   (D) 2   (E) 5

GO ON TO THE NEXT PAGE.

9. Which of the following gives the length of the curve defined by the parametric equations $x(t) = \dfrac{t^2}{2}$

and $y(t) = \dfrac{t^3}{3}$ from $t = 0$ to $t = 1$?

(A) $\displaystyle\int_0^1 \sqrt{1 + t^2}\ dt$

(B) $\displaystyle\int_0^1 \sqrt{1 + 4t^2}\ dt$

(C) $\displaystyle\int_0^1 \sqrt{1 + t^4}\ dt$

(D) $\displaystyle\int_0^1 \sqrt{t^2 + t^4}\ dt$

(E) $\displaystyle\int_0^1 \sqrt{\dfrac{t^4}{4} + \dfrac{t^6}{9}}\ dt$

GO ON TO THE NEXT PAGE.

10. Let f be the function given by $f(x) = 3 - 2x.$ If g is a function with derivative given by $g'(x) = f(x)f'(x)(x - 3),$ on what intervals is g increasing?

(A) $\left(-\infty, \dfrac{3}{2}\right]$ and $[3, \infty)$

(B) $\left(-\infty, \dfrac{3}{2}\right]$ only

(C) $\left[\dfrac{3}{2}, 3\right]$ only

(D) $\left[\dfrac{3}{2}, \infty\right)$

(E) $[3, \infty)$ only

x	0	1	2	3	4	5	6
$f(x)$	0	5	2	-1	-2	0	3

11. The function f is continuous on the closed interval $[0, 6]$ and has values as shown in the table above. Using the intervals $[0, 2]$, $[2, 4]$, and $[4, 6]$, what is the approximation of $\int_0^6 f(x)\, dx$ obtained from a midpoint Riemann sum?

(A) 0   (B) 3   (C) 4   (D) 6   (E) 8

12. $\int x^2 \sin x\, dx =$

(A) $-2x\cos x + \int x^2 \cos x\, dx$

(B) $-x^2 \cos x + 2\int x\cos x\, dx$

(C) $x^2 \sin x + 2\int x\cos x\, dx$

(D) $-\dfrac{1}{3}x^3 \cos x + C$

(E) $\dfrac{1}{3}x^3 \sin x - x^2 \cos x + C$

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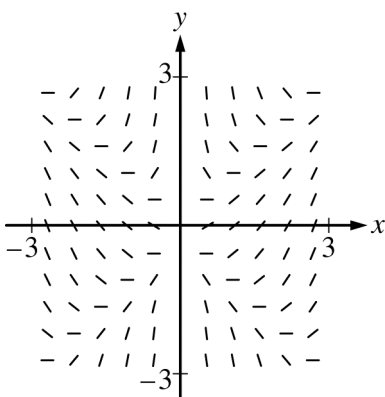
13. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = \dfrac{x^2 - y^2}{x}$

(B) $\dfrac{dy}{dx} = \dfrac{x^2 - y^2}{y}$

(C) $\dfrac{dy}{dx} = x^2 - y^2$

(D) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{x}$

(E) $\dfrac{dy}{dx} = x^2 + y^2$

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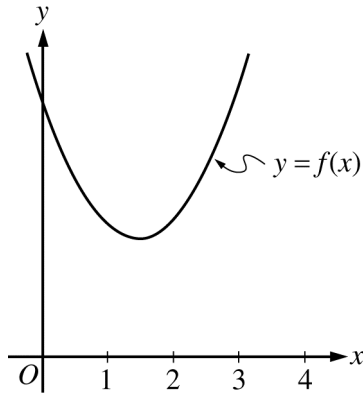
14. The figure above shows the graph of a function f. Which of the following could be the second-degree Taylor polynomial for f about $x = 2$?

(A) $2 - x - x^2$

(B) $2 + x - x^2$

(C) $2 - (x - 2) + (x - 2)^2$

(D) $2 + (x - 2) - (x - 2)^2$

(E) $2 + (x - 2) + (x - 2)^2$

GO ON TO THE NEXT PAGE.

15. What are all values of p for which the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{2p}+n}$ diverges?

(A) $p \le \dfrac{1}{2}$

(B) $p < \dfrac{1}{2}$ only

(C) $p \ge \dfrac{1}{2}$

(D) $p > \dfrac{1}{2}$ only

(E) The series diverges for all p.

16. If $\cos(xy) = y - 1$, then the value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{2}$ and $y = 1$ is

(A) $\dfrac{-2}{2-\pi}$ (B) $\dfrac{-2}{2+\pi}$ (C) 0 (D) $\dfrac{2}{2-\pi}$ (E) $\dfrac{2}{2+\pi}$

GO ON TO THE NEXT PAGE.

17. Let f be the function defined by $f(x) = \dfrac{1}{x}$. What is the average value of f on the interval $[4, 6]$?

(A) $-\dfrac{1}{24}$ (B) $\dfrac{5}{24}$ (C) $\dfrac{1}{2}\ln\dfrac{3}{2}$ (D) $\ln\dfrac{3}{2}$ (E) $\dfrac{1}{2}\ln 2$

18. Let P be the second-degree Taylor polynomial for e^{-2x} about $x = 3$. What is the slope of the line tangent to the graph of P at $x = 3$?

(A) $-2e^{-6}$ (B) e^{-6} (C) $2e^{-6}$ (D) $4e^{-6}$ (E) $10e^{-6}$

GO ON TO THE NEXT PAGE.

19. What are all values of x for which $\int_x^2 t^3\, dt$ is equal to 0 ?

    (A) -2 only   (B) 0 only   (C) 2 only   (D) -2 and 2 only   (E) -2, 0, and 2

20. A curve C is defined by the parametric equations $x(t) = 3 + t^2$ and $y(t) = t^3 + 5t$. Which of the following is an equation of the line tangent to the graph of C at the point where $t = 1$?

    (A) $y = \dfrac{1}{4}x + 5$

    (B) $y = 4x - 10$

    (C) $y = 4x + 6$

    (D) $y = 8x - 26$

    (E) $y = 8x + 6$

GO ON TO THE NEXT PAGE.

21. The power series $\sum_{n=1}^{\infty} \dfrac{(x-5)^n}{2^n n^2}$ has radius of convergence 2. At which of the following values of x can the alternating series test be used with this series to verify convergence at x ?

   (A) 6   (B) 4   (C) 2   (D) 0   (E) −1

22. Let f be the function given by $f(x) = \dfrac{1}{2+x}$. What is the coefficient of x^3 in the Taylor series for f about $x = 0$?

   (A) $-\dfrac{3}{8}$ (B) $-\dfrac{1}{8}$ (C) $-\dfrac{1}{16}$ (D) $\dfrac{1}{24}$ (E) $\dfrac{1}{16}$

GO ON TO THE NEXT PAGE.

23. $\displaystyle \int \frac{3}{x(x+1)}\, dx =$

(A) $3\tan^{-1} x + C$

(B) $3\ln\left|x^2 + x\right| + C$

(C) $\dfrac{3}{2x+1}\ln\left|x^2 + x\right| + C$

(D) $3\ln\left|\dfrac{x}{x+1}\right| + C$

(E) $3\ln\left|\dfrac{x+1}{x}\right| + C$

GO ON TO THE NEXT PAGE.

24. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$

III. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

(A) I only   (B) II only   (C) III only   (D) I and II only   (E) I, II, and III

25. $\displaystyle\lim_{h\to 0} \frac{e^{-1-h} - e^{-1}}{h}$ is

(A) -1 (B) $\dfrac{-1}{e}$ (C) 0 (D) $\dfrac{1}{e}$ (E) nonexistent

GO ON TO THE NEXT PAGE.

26. Let f be the function given by $f(x) = x^3 - 2x^2 + 5x - 16$. For what value of x in the closed interval $[0, 5]$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

(A) 0   (B) $\dfrac{5}{3}$ (C) $\dfrac{5}{2}$ (D) 3   (E) 5

GO ON TO THE NEXT PAGE.

27. If R is the unbounded region between the graph of $y = \dfrac{1}{x(\ln x)^2}$ and the x-axis for $x \geq 3$, then the area of R is

(A) $-\dfrac{1}{\ln 3}$
(B) $\ln(\ln 3)$
(C) $\dfrac{1}{\ln 3}$
(D) $\dfrac{3}{(\ln 3)^3}$
(E) infinite

GO ON TO THE NEXT PAGE.

28. A particle travels along the x-axis so that at time $t \geq 0$ its velocity is given by $v(t) = t^2 - 6t + 8$. What is the total distance the particle travels from $t = 0$ to $t = 3$?

(A) 1 (B) 6 (C) $\dfrac{20}{3}$ (D) $\dfrac{22}{3}$ (E) 8

END OF PART A OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1)   The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

76. A music group expects to sell a new compact disc (CD) at the rate $R(t) = 20{,}000e^{-0.12t}$ CDs per week, where t denotes the number of weeks since the CD was first released. To the nearest thousand, how many CDs are expected to be sold during the first 12 weeks after the release?

(A) 5,000   (B) 11,000   (C) 57,000   (D) 127,000   (E) 240,000

77. Let f be a function that is continuous on the closed interval $[1, 3]$ with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?

(A) $10 \le f(2) \le 18$

(B) f is increasing on the interval $[1, 3]$.

(C) $f(x) = 17$ has at least one solution in the interval $[1, 3]$.

(D) $f'(x) = 8$ has at least one solution in the interval $(1, 3)$.

(E) $\int_{1}^{3} f(x)\,dx > 20$

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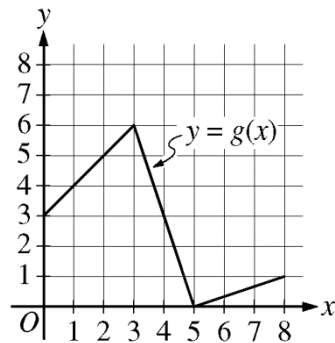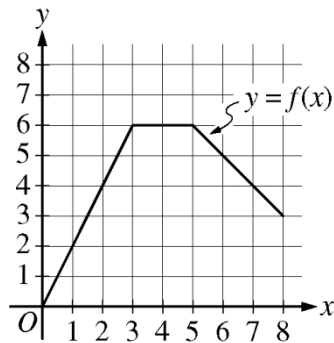
78. The graphs of the piecewise linear functions f and g are shown above. If the function h is defined by $h(x) = f(x)g(x)$, then $h'(2)$ is

(A) 2   (B) 13   (C) 14   (D) 20   (E) nonexistent

79. The number of people who have entered a museum on a certain day is modeled by a function $f(t)$, where t is measured in hours since the museum opened that day. The number of people who have left the museum since it opened that same day is modeled by a function $g(t)$. If $f'(t) = 380\left(1.02^t\right)$ and

$g'(t) = 240 + 240\sin\left(\dfrac{\pi(t-4)}{12}\right)$, at what time t, for $1 \le t \le 11$, is the number of people in the museum at a maximum?

(A) 1   (B) 7.888   (C) 9.446   (D) 10.974   (E) 11

GO ON TO THE NEXT PAGE.

x	0	1	2	3
$f(x)$	5	2	3	6
$f'(x)$	−3	1	3	4

80. The derivative of the function f is continuous on the closed interval $[0, 4]$. Values of f and f' for selected

values of x are given in the table above. If $\int_0^4 f'(t)\, dt = 8$, then $f(4) =$

(A) 0   (B) 3   (C) 5   (D) 10   (E) 13

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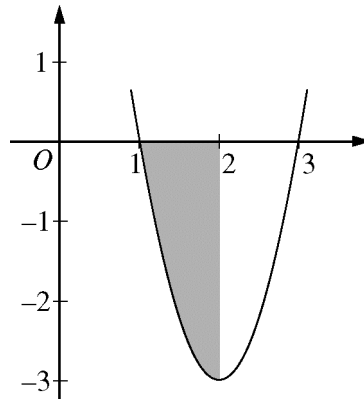
Graph of f

81. The figure above shows the graph of the function f. If $g(x) = \int_1^x f(t)\, dt$ and the shaded region has an area of 2, what is the value of $g(2)$?

   (A) −3   (B) −2   (C) 0   (D) 1   (E) 2

---

82. A particle moves along the x-axis so that at time $t \geq 0$ the position of the particle is given by $x(t) = 0.5t^4 - 1.5t^3 - 2t^2 + 6t - 1$. What is the velocity of the particle at the first instance the particle is at the origin?

   (A) −4.071   (B) −2.048   (C) 0   (D) 5.153   (E) 6

GO ON TO THE NEXT PAGE.

y

O | 1 2 3 4 5 6 7 8 → x
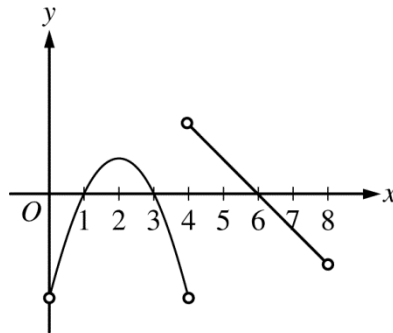
Graph of f'

83. The graph of f', the derivative of a function f, is shown above. Which of the following could be the graph of f?

I.

y

O | 1 2 3 4 5 6 7 8 → x

II.

y

O | 1 2 3 4 5 6 7 8 → x

III.

y

O | 1 2 3 4 5 6 7 8 → x
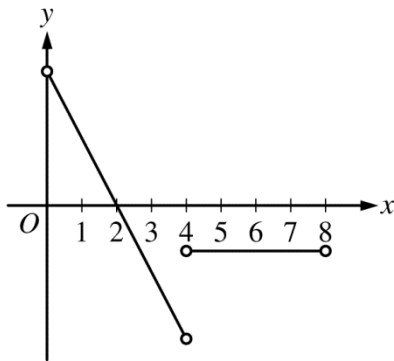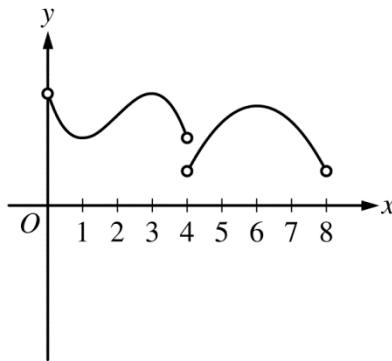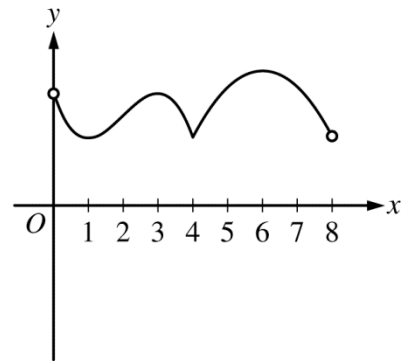
(A) I only

(B) II only

(C) III only

(D) I and II only

(E) II and III only

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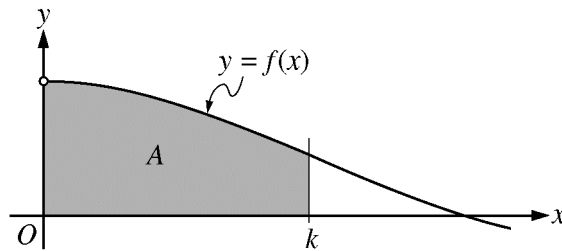
84. The figure above shows the region A, which is bounded by the x- and y-axes, the graph of $f(x) = \dfrac{\sin x}{x}$

for $x > 0$, and the vertical line $x = k$. If k increases at a rate of $\dfrac{\pi}{4}$ units per second, how fast is the area of

region A increasing when $k = \dfrac{\pi}{6}$?

(A) 0　　　(B) $\dfrac{3}{4}$　　　(C) $\dfrac{3}{\pi}$　　　(D) $\dfrac{\sqrt{3}}{2}$　　　(E) $2\sqrt{3}$

GO ON TO THE NEXT PAGE.

85. The function f is defined on the open interval $0.4 < x < 2.4$ and has first derivative f' given by $f'(x) = \sin\left(x^2\right)$. Which of the following statements are true?

   I. f has a relative maximum on the interval $0.4 < x < 2.4$.

   II. f has a relative minimum on the interval $0.4 < x < 2.4$.

   III. The graph of f has two points of inflection on the interval $0.4 < x < 2.4$.

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

GO ON TO THE NEXT PAGE.

86. The position of a particle moving along the x-axis is given by a twice-differentiable function $x(t)$. If $x(2) < 0$, $x'(2) < 0$, and $x''(2) < 0$, which of the following statements must be true about the particle at time $t = 2$?

   (A) The particle is moving toward the origin at a decreasing speed.

   (B) The particle is moving toward the origin at an increasing speed.

   (C) The particle is moving away from the origin at a decreasing speed.

   (D) The particle is moving away from the origin at an increasing speed.

   (E) The particle is moving away from the origin at a constant speed.

87. If $0 \le b \le 2$, for what value of b is $\int_0^b \cos\left(e^x\right) dx$ a minimum?

   (A) 0

   (B) 0.452

   (C) 1.145

   (D) 1.550

   (E) 2

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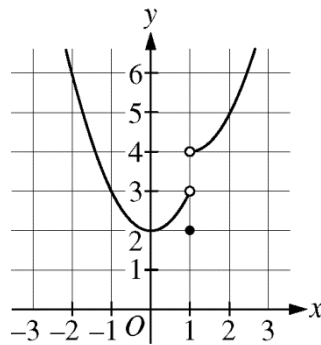
Graph of f

88. The graph of the function f is shown in the figure above. The value of $\lim_{x \to 0} f\left(1 - x^2\right)$ is

(A) 1   (B) 2   (C) 3   (D) 4   (E) nonexistent

89. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2 \sin t$ for $0 \le t \le 2$. What is the total distance traveled by the particle between $t = 0$ and $t = 2$?

(A) −3.637

(B) −1.973

(C)   1.531

(D)   2.539

(E)   3.637

GO ON TO THE NEXT PAGE.

90. A cup has the shape of a right circular cone. The height of the cup is 12 cm, and the radius of the opening is 3 cm. Water is poured into the cup at a constant rate of $2 \text{ cm}^3/\text{sec}$. What is the rate at which the water level is rising when the depth of the water in the cup is 5 cm? (The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(A) $\dfrac{32}{25\pi}$ cm/sec

(B) $\dfrac{96}{125\pi}$ cm/sec

(C) $\dfrac{2}{3\pi}$ cm/sec

(D) $\dfrac{2}{9\pi}$ cm/sec

(E) $\dfrac{1}{200\pi}$ cm/sec

GO ON TO THE NEXT PAGE.

91. The number of students in a school who have heard a rumor at time t hours is modeled by the function P, the solution to a logistic differential equation. At noon, 50 of the school's 500 students have heard the rumor. Also at noon, P is increasing at a rate of 20 students per hour. Which of the following could be the logistic differential equation?

(A) $\dfrac{dP}{dt} = \dfrac{1}{1125} P(500 - P)$

(B) $\dfrac{dP}{dt} = \dfrac{1}{480} P(500 - P)$

(C) $\dfrac{dP}{dt} = \dfrac{1}{192} P(500 - P)$

(D) $\dfrac{dP}{dt} = \dfrac{2}{45} P(500 - P)$

(E) $\dfrac{dP}{dt} = \dfrac{5}{48} P(500 - P)$

GO ON TO THE NEXT PAGE.

92. Let f be a positive, continuous, decreasing function. If $\int_1^\infty f(x)\, dx = 5,$ which of the following statements about the series $\displaystyle\sum_{n=1}^\infty f(n)$ must be true?

(A) $\displaystyle\sum_{n=1}^\infty f(n) = 0$

(B) $\displaystyle\sum_{n=1}^\infty f(n)$ converges, and $\displaystyle\sum_{n=1}^\infty f(n) < 5.$

(C) $\displaystyle\sum_{n=1}^\infty f(n) = 5$

(D) $\displaystyle\sum_{n=1}^\infty f(n)$ converges, and $\displaystyle\sum_{n=1}^\infty f(n) > 5.$

(E) $\displaystyle\sum_{n=1}^\infty f(n)$ diverges.

GO ON TO THE NEXT PAGE.

B   B   B   B   B   B   B   B   B

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.
_____

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**

- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR
  ANSWER SHEET**

- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET
  AND PLACED IT ON YOUR ANSWER SHEET**

**AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND
ANSWER QUESTIONS 93–96.**

Section II: Free-Response Questions

This is the free-response section of the 2016 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

   First letter of your first name

2. Date of birth

   Month   Day   Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

   No, I do not grant the College Board these rights.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_{1}^{5} x^2 \, dx$ may not be written as fnInt(X², X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4LBP6-S

68

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. The curve C in the xy-plane is given parametrically by $(x(t), y(t))$, where $\dfrac{dx}{dt} = t \sin\left(\dfrac{2t}{3}\right)$ and $\dfrac{dy}{dt} = \cos\left(\dfrac{t^2}{4}\right)$

   for $1 \le t \le 6$.

   (a) Find the slope of the line tangent to the curve C at the point where $t = 3$.

   (b) For $1 \le t \le 6$, what is the value of t at which the line tangent to the curve C is vertical?

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Continue problem 1 on page 5.

-4-

(c) Find the length of the curve C for $1 \leq t \leq 6$.

(d) Given $y(1) = 2$, find $y(3)$.

t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

2. A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \leq t \leq 12$. Values of $G(t)$ are given in the table above for selected values of t.

(a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

(b) Approximate the value of $\int_0^{12} G(t)\, dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\int_0^{12} G(t)\, dt$ in the context of this problem.

(c) One salesperson believes that, starting with 2400 games per week at time $t = 12,$ the rate at which games will be sold will increase at a constant rate of 100 games per week per week. Based on this model, how many total games will be sold in the 8 weeks between time $t = 12$ and $t = 20$?

(d) Another salesperson believes the best model for the rate at which games will be sold in the 8 weeks between time $t = 12$ and $t = 20$ is $M(t) = 2400e^{-0.01(t-12)^2}$ games per week. Based on this model, how many total games, to the nearest whole number, will be sold during this period?

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END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED
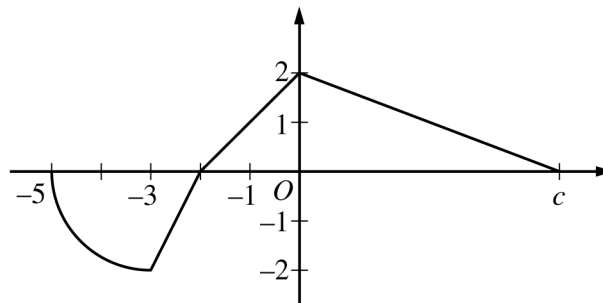
Graph of f

3. The function f is defined on the interval $-5 \le x \le c,$ where $c > 0$ and $f(c) = 0.$ The graph of $f,$ which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.

(a) Find the average rate of change of f over the interval $[-5, 0].$ Show the computations that lead to your answer.

(b) For $-5 \le x \le c,$ let g be the function defined by $g(x) = \int_{-1}^{x} f(t) \, dt.$ Find the x-coordinate of each point of inflection of the graph of $g.$ Justify your answer.

-14-

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(c) Find the value of c for which the average value of f over the interval $-5 \leq x \leq c$ is $\frac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\dfrac{x}{2}\right)$. Find $h'(6)$ in terms of c.

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4. Let R be the region bounded above by the graph of $y = e^{-x}$, below by the x-axis, on the left by the y-axis, and on the right by the vertical line $x = m$, where $m > 0$ is a constant.

(a) Find the area of R in terms of m.

(b) Region R is revolved around the horizontal line $y = -3$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

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(c) Find the value of the real number $k > 1$ such that $\int_0^\infty k^{-x}\, dx = 2$, or show that no such k exists.

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-17-

5. For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\dfrac{dW}{dt} = \dfrac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius ($^\circ$C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

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(c) The temperature in the kitchen remains constant at $20°C$ for $0 \le t \le 24.$ The cost of operating the refrigerator accumulates at the rate of $0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

GO ON TO THE NEXT PAGE.

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6. Let f be a function that has derivatives of all orders for all real numbers, and let
$3 - 6(x-1)^2 + 5(x-1)^3 - 2(x-1)^4$ be the fourth-degree Taylor polynomial for the function f about $x = 1$.

(a) Find $f'''(1)$.

(b) Does the graph of f have a relative maximum, a relative minimum, or neither at $x = 1$? Give a reason for your answer.

(c) Let g be the function defined by $g(x) = \int_1^x f(t)\, dt$. Find the third-degree Taylor polynomial for g about $x = 1$, and use it to approximate $g(1.1)$.

(d) Let h be the function defined by $h(x) = f'(x)$. Find the third-degree Taylor polynomial for h about $x = 1$. Verify that the graph of h does not have a point of inflection at $x = 1$.

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STOP

END OF EXAM

_____

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT <u>AND</u> BACK COVERS OF THE SECTION II BOOKLET.**

- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**

- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to
the multiple-choice questions in this exam.

Answer Key for AP Calculus BC
Practice Exam, Section I

Question 1: B

Question 2: E

Question 3: D

Question 4: D

Question 5: C

Question 6: B

Question 7: C

Question 8: D

Question 9: D

Question 10: A

Question 11: E

Question 12: B

Question 13: A

Question 14: E

Question 15: A

Question 16: B

Question 17: C

Question 18: A

Question 19: D

Question 20: B

Question 21: B

Question 22: C

Question 23: D

Question 24: D

Question 25: B

Question 26: D

Question 27: C

Question 28: D

Question 76: D

Question 77: C

Question 78: C

Question 79: B

Question 80: E

Question 81: B

Question 82: D

Question 83: E

Question 84: B

Question 85: D

Question 86: D

Question 87: D

Question 88: C

Question 89: D

Question 90: A

Question 91: A

Question 92: D

Free-Response Scoring Guidelines

The following contains the scoring guidelines
for the free-response questions in this exam.

Question 1

The curve C in the xy-plane is given parametrically by $(x(t), y(t))$, where $\dfrac{dx}{dt} = t\sin\left(\dfrac{2t}{3}\right)$ and $\dfrac{dy}{dt} = \cos\left(\dfrac{t^2}{4}\right)$

for $1 \le t \le 6$.

(a) Find the slope of the line tangent to the curve C at the point where $t = 3$.

(b) For $1 \le t \le 6$, what is the value of t at which the line tangent to the curve C is vertical?

(c) Find the length of the curve C for $1 \le t \le 6$.

(d) Given $y(1) = 2$, find $y(3)$.

(a) Slope $= \dfrac{y'(3)}{x'(3)} = \dfrac{\cos(9/4)}{3\sin(2)} = -0.230$

$2 : \begin{cases} 1 : \text{expression for the slope} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dx}{dt} = t\sin\left(\dfrac{2t}{3}\right) = 0 \Rightarrow t = \dfrac{3\pi}{2} \approx 4.712$

$2 : \begin{cases} 1 : \text{sets } \dfrac{dx}{dt} = 0 \\ 1 : \text{answer} \end{cases}$

(c) Length $= \displaystyle\int_1^6 \sqrt{(x'(t))^2 + (y'(t))^2}\ dt = 10.569\ \text{(or 10.568)}$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $y(3) = 2 + \displaystyle\int_1^3 y'(t)\ dt = 2.805\ \text{(or 2.804)}$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{initial condition} \\ 1 : \text{answer} \end{cases}$

Question 2

t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \le t \le 12$. Values of $G(t)$ are given in the table above for selected values of t.

(a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

(b) Approximate the value of $\int_0^{12} G(t)\, dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\int_0^{12} G(t)\, dt$ in the context of this problem.

(c) One salesperson believes that, starting with 2400 games per week at time $t = 12$, the rate at which games will be sold will increase at a constant rate of 100 games per week per week. Based on this model, how many total games will be sold in the 8 weeks between time $t = 12$ and $t = 20$?

(d) Another salesperson believes the best model for the rate at which games will be sold in the 8 weeks between time $t = 12$ and $t = 20$ is $M(t) = 2400e^{-0.01(t-12)^2}$ games per week. Based on this model, how many total games, to the nearest whole number, will be sold during this period?

(a) $G'(8) \approx \dfrac{G(10) - G(6)}{10 - 6} = \dfrac{2100 - 900}{10 - 6} = 300$ games per week per week

1 : approximation

(b) $\int_0^{12} G(t)\, dt \approx 3 \cdot G(3) + 3 \cdot G(6) + 4 \cdot G(10) + 2 \cdot G(12)$

$= 3 \cdot 450 + 3 \cdot 900 + 4 \cdot 2100 + 2 \cdot 2400$

$= 17250$

$\int_0^{12} G(t)\, dt$ represents the total number of games sold over the 12-week period $0 \le t \le 12$.

$3 : \begin{cases} 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

(c) The rate of sales of the game for $12 \le t \le 20$ is
$R(t) = 2400 + 100(t - 12)$.

Based on this model, the total number of games that will be sold between time $t = 12$ and $t = 20$ is $\int_{12}^{20} R(t)\, dt = 22400$.

$3 : \begin{cases} 1 : \text{rate function} \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\int_{12}^{20} M(t)\, dt = 15784.07655$
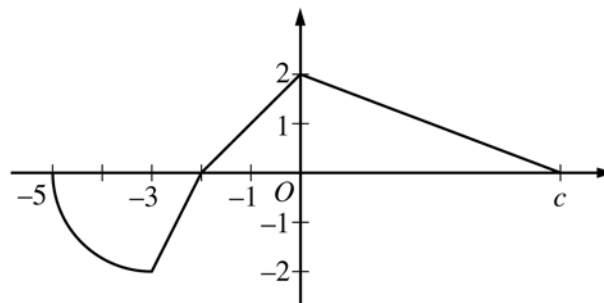
Based on this model, the total number of games that will be sold between time $t = 12$ and $t = 20$ is 15784.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Question 3

The function f is defined on the interval $-5 \le x \le c$, where $c > 0$ and $f(c) = 0$. The graph of f, which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.

Graph of f

(a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

(b) For $-5 \le x \le c$, let g be the function defined by

$g(x) = \int_{-1}^{x} f(t)\, dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\dfrac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\dfrac{x}{2}\right)$. Find $h'(6)$ in terms of c.

(a) The average rate of change of f over the interval $[-5, 0]$ is

$\dfrac{f(0) - f(-5)}{0 - (-5)} = \dfrac{2}{5}$.

1 : answer

(b) $g'(x) = f(x)$

The graph of g has a point of inflection at $x = -3$ because $g' = f$ changes from decreasing to increasing at this point.

The graph of g has a point of inflection at $x = 0$ because $g' = f$ changes from increasing to decreasing at this point.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -3 \text{ and } x = 0 \\ 1 : \text{justification} \end{cases}$

(c) $\dfrac{1}{c+5}\displaystyle\int_{-5}^{c} f(x)\, dx = \dfrac{1}{2}$

$\dfrac{1}{c+5}(-\pi + (-1) + 2 + c) = \dfrac{1}{2}$

$c = 3 + 2\pi$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

(d) $h'(x) = \dfrac{1}{2} f'\left(\dfrac{x}{2}\right)$

$h'(6) = \dfrac{1}{2} f'(3) = \dfrac{1}{2} \cdot \dfrac{-2}{c} = -\dfrac{1}{c}$

$2 : \begin{cases} 1 : h'(x) \\ 1 : \text{answer} \end{cases}$

Question 4

Let R be the region bounded above by the graph of $y = e^{-x}$, below by the x-axis, on the left by the y-axis, and on the right by the vertical line $x = m$, where $m > 0$ is a constant.

(a) Find the area of R in terms of m.

(b) Region R is revolved around the horizontal line $y = -3$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) Find the value of the real number $k > 1$ such that $\int_0^\infty k^{-x}\, dx = 2$, or show that no such k exists.

(a) Area $= \int_0^m e^{-x}\, dx = -\left[e^{-x}\right]_0^m = -e^{-m} + 1$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(b) Volume $= \pi \int_0^m \left[\left(e^{-x} + 3\right)^2 - 3^2\right] dx$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

(c) $\displaystyle\lim_{b\to\infty} \int_0^b k^{-x}\, dx = -\frac{1}{\ln k} \lim_{b\to\infty} \left[k^{-x}\right]_0^b$

$\qquad\qquad\quad = -\frac{1}{\ln k} \lim_{b\to\infty} \left(\frac{1}{k^b} - 1\right) = \frac{1}{\ln k}$

$\dfrac{1}{\ln k} = 2 \Rightarrow k = e^{1/2}$

$$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit} \\ 1 : \text{answer} \end{cases}$$

Question 5

For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\dfrac{dW}{dt} = \dfrac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

(a) $\left.\dfrac{dW}{dt}\right|_{(t,\,W)=(0,\,3)} = \dfrac{3\cos 0}{2(3)} = \dfrac{1}{2}$

An equation for the tangent line is $y = \dfrac{1}{2}t + 3$.

$W(0.4) \approx \dfrac{1}{2}(0.4) + 3 = 3.2°C$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $2W \; dW = 3\cos t \; dt$

$\displaystyle\int 2W \; dW = \int 3\cos t \; dt$

$W^2 = 3\sin t + C$

$3^2 = 3\sin 0 + C \Rightarrow C = 9$

$W^2 = 3\sin t + 9$

Since $W(0) = 3$, $W = \sqrt{3\sin t + 9}$ for $0 \le t \le 24$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \quad \text{and uses initial condition} \\ 1 : \text{solves for } W \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

(c) $0.001\displaystyle\int_0^{24}(20 - W(t))\, dt$ dollars

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

Question 6

Let f be a function that has derivatives of all orders for all real numbers, and let
$3 - 6(x-1)^2 + 5(x-1)^3 - 2(x-1)^4$ be the fourth-degree Taylor polynomial for the function f about $x = 1$.

(a) Find $f'''(1)$.

(b) Does the graph of f have a relative maximum, a relative minimum, or neither at $x = 1$? Give a reason for your answer.

(c) Let g be the function defined by $g(x) = \int_1^x f(t)\, dt$. Find the third-degree Taylor polynomial for g about $x = 1$, and use it to approximate $g(1.1)$.

(d) Let h be the function defined by $h(x) = f'(x)$. Find the third-degree Taylor polynomial for h about $x = 1$. Verify that the graph of h does not have a point of inflection at $x = 1$.

(a) $5 = \dfrac{f'''(1)}{6} \Rightarrow f'''(1) = 30$

1 : answer

(b) $f'(1) = 0$

$-6 = \dfrac{f''(1)}{2} \Rightarrow f''(1) = -12 < 0$

Therefore, f has a relative maximum at $x = 1$.

$2 : \begin{cases} 1 : f'(1) = 0 \\ 1 : \text{answer with reason} \end{cases}$

(c) Let G_3 be the third-degree Taylor polynomial for g about $x = 1$.

$G_3(x) = \int_1^x \left(3 - 6(t-1)^2\right) dt$

$\qquad = \left[3(t-1) - 2(t-1)^3 \right]_1^x$

$\qquad = 3(x-1) - 2(x-1)^3$

$g(1.1) \approx G_3(1.1) = 3(0.1) - 2(0.1)^3 = 0.298$

$3 : \begin{cases} 2 : \text{Taylor polynomial} \\ 1 : \text{approximation} \end{cases}$

(d) Let H_3 be the third-degree Taylor polynomial for h about $x = 1$.

$H_3(x) = \dfrac{d}{dx}\left[3 - 6(x-1)^2 + 5(x-1)^3 - 2(x-1)^4 \right]$

$\qquad = -12(x-1) + 15(x-1)^2 - 8(x-1)^3$

$15 = \dfrac{h''(1)}{2} \Rightarrow h''(1) = 30$

Since $h''(1) \neq 0$, the graph of h does not have a point of inflection at $x = 1$.

$3 : \begin{cases} 2 : \text{Taylor polynomial} \\ 1 : \text{reason} \end{cases}$

Scoring Worksheets

The following provides scoring worksheets and conversion tables used for calculating a composite score of the exam.

2016 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

_____ × 1.2000 = _____
Number Correct   Weighted Section I Score
(out of 45)   (Do not round)

Section II: Free Response

Question 1   _____ × 1.0000 = _____
                        (out of 9)   (Do not round)

Question 2   _____ × 1.0000 = _____
                        (out of 9)   (Do not round)

Question 3   _____ × 1.0000 = _____
                        (out of 9)   (Do not round)

Question 4   _____ × 1.0000 = _____
                        (out of 9)   (Do not round)

Question 5   _____ × 1.0000 = _____
                        (out of 9)   (Do not round)

Question 6   _____ × 1.0000 = _____
                        (out of 9)   (Do not round)

Sum = _____
Weighted
Section II
Score
(Do not round)

Composite Score

_____ + _____ = _____
Weighted   Weighted   Composite Score
Section I Score   Section II Score   (Round to nearest
                                                            whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
64-108	5
54-63	4
40-53	3
34-39	2
0-33	1

2016 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice
Questions (4, 6, 8, 10-11, 13, 16-17, 19, 25-26, 28, 76-90)

_____ × 1.0000 = _____
Number Correct   Weighted Section I Score
(out of 27)   (Do not round)

Section II: Free Response

Question 2   _____ × 1.0000 = _____
          (out of 9)   (Do not round)

Question 3   _____ × 1.0000 = _____
          (out of 9)   (Do not round)

Question 5   _____ × 1.0000 = _____
          (out of 9)   (Do not round)

Sum = _____
Weighted
Section II
Score
(Do not round)

Composite Score

_____ + _____ = _____
Weighted   Weighted   Composite Score
Section I Score   Section II Score   (Round to nearest
                                           whole number)

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Score
31-54	5
25-30	4
18-24	3
15-17	2
0-14	1

Question Descriptors and Performance Data

The following contains tables showing the content assessed,
the correct answer, and how AP students performed on each question.

2016 AP Calculus BC
Question Descriptors and Performance Data

Multiple-Choice Questions

Question	Topic	Key	% Correct
1	Derivatives	B	62
2	Derivatives	E	88
3	Polynomial Approximation And Series	D	71
4	Derivatives	D	81
5	Integrals	C	84
6	Derivatives	B	77
7	Derivatives	C	75
8	Integrals	D	74
9	Integrals	D	70
10	Derivatives	A	74
11	Integrals	E	78
12	Integrals	B	82
13	Derivatives	A	66
14	Polynomial Approximation And Series	E	74
15	Polynomial Approximation And Series	A	45
16	Derivatives	B	51
17	Integrals	C	75
18	Polynomial Approximation And Series	A	52
19	Integrals	D	80
20	Derivatives	B	80
21	Polynomial Approximation And Series	B	62
22	Polynomial Approximation And Series	C	48
23	Integrals	D	61
24	Polynomial Approximation And Series	D	49
25	Derivatives	B	49
26	Derivatives	D	46
27	Integrals	C	41
28	Integrals	D	22
76	Integrals	D	81
77	Functions, Graphs, And Limits	C	80
78	Derivatives	C	85
79	Derivatives	B	61
80	Integrals	E	72
81	Integrals	B	62
82	Derivatives	D	47
83	Derivatives	E	77
84	Integrals	B	44
85	Derivatives	D	69

2016 AP Calculus BC

Question Descriptors and Performance Data

Question	Topic	Key	% Correct
86	Derivatives	D	51
87	Integrals	D	73
88	Functions, Graphs, And Limits	C	9
89	Integrals	D	53
90	Derivatives	A	44
91	Integrals	A	47
92	Polynomial Approximation And Series	D	25

AP Calculus BC

The College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.