

From the 2018 Administration

AP[®] CollegeBoard

AP Calculus BC

Practice Exam

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Note: This publication shows the page numbers that appeared in the **2017–18 AP Exam Instructions** book and in the actual exam. This publication was not repaginated to begin with page 1.

Exam Instructions

The following contains instructions taken from the **2017–18 AP Exam Instructions** book.

AP Calculus AB/BC Exam

Regularly Scheduled Exam Date: Tuesday morning, May 15, 2018

Late-Testing Exam Date: Friday morning, May 25, 2018

Section I	Total Time: 1 hour and 45 minutes	Part A:	Time: 1 hour
	Number of Questions: 45 <i>(The number of questions may vary slightly depending on the form of the exam.)</i>	Number of Questions: 30	<i>No calculator allowed</i>
	Percent of Total Score: 50%	Part B:	Time: 45 minutes
	Writing Instrument: Pencil required	Number of Questions: 15	<i>Graphing calculator required</i>
Section II	Total Time: 1 hour and 30 minutes	Part A:	Time: 30 minutes
	Number of Questions: 6	Number of Questions: 2	Percent of Section II Score: 33.33%
	Percent of Total Score: 50%		<i>Graphing calculator required</i>
	Writing Instrument: Either pencil or pen with black or dark blue ink	Part B:	Time: 1 hour
	Note: For Section II, if students finish Part A before the end of the timed 30 minutes for Part A, they cannot begin working on Part B. Students must wait until the beginning of the timed 1 hour for Part B. However, during the timed portion for Part B, students may work on the questions in Part A without the use of a calculator.	Number of Questions: 4	Percent of Section II Score: 66.67%
			<i>No calculator allowed</i>

Before Distributing Exams: Check that the title on all exam covers is *Calculus AB* or *Calculus BC*. Be sure to distribute the correct exam—AB or BC—to the students. If there are any exam booklets with a different title, contact the AP coordinator immediately.

What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- 2017-18 AP Coordinator's Manual*
- This book—*2017-18 AP Exam Instructions*
- AP Exam Seating Chart template
- School Code and Homeschool/Self-Study Codes
- Extra graphing calculators
- Pencil sharpener
- Container for students' electronic devices (if needed)
- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
 - “Exam in Progress”
 - “Cell phones are prohibited during the test administration, including breaks”

SEATING POLICY FOR AP CALCULUS AB AND CALCULUS BC EXAMS

Testing Window	Exams Administered at Schools in the United States, Canada, Puerto Rico, and the U.S. Virgin Islands	Exams Administered at Schools Outside the United States, Canada, Puerto Rico, and the U.S. Virgin Islands
Regularly Scheduled Exams	Students must be seated no less than 4 feet apart.	Students must be seated no less than 5 feet apart.
Late-Testing Exams	Students must be seated no less than 5 feet apart.	

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 52 of the *2017-18 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 51 of the *AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

› **Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.**

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time. Remember, you must complete a seating chart for this exam. See pages 303–304 for a seating chart template and instructions. See the *2017-18 AP Coordinator's Manual* for exam seating requirements (pages 55–58).

If you are giving the regularly scheduled exam, say:

It is Tuesday morning, May 15, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Friday morning, May 25, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the AP Calculus AB Exam, say:

Look at your exam packet and confirm that the exam title is “AP Calculus AB.”
Raise your hand if your exam packet contains any title other than “AP Calculus AB,”
and I will help you.

If you are giving the AP Calculus BC Exam, say:

Look at your exam packet and confirm that the exam title is “AP Calculus BC.”
Raise your hand if your exam packet contains any title other than “AP Calculus BC,”
and I will help you.

If you are giving both the AP Calculus AB Exam and AP Calculus BC Exam, say:

Look at your exam packet and confirm that the exam title is “AP Calculus AB” or
“AP Calculus BC,” depending upon which exam you are taking today. Raise your
hand if your exam packet contains any other title and I will help you.

Once you confirm that all students have the correct exam, say:

In a moment, you will open the exam packet. By opening this packet, you
agree to all of the AP Program’s policies and procedures outlined in the
2017-18 Bulletin for AP Students and Parents.

You may now remove the shrinkwrap from your exam packet and take out the
Section I booklet, but do not open the booklet or the shrinkwrapped Section II
materials. Put the white seals aside. . . .

Carefully remove the AP Exam label found near the top left of your exam
booklet cover. Place it on page 1 of your answer sheet on the light blue box near
the top right corner that reads “AP Exam Label.”

If students accidentally place the exam label in the space for the number label or vice
versa, advise them to leave the labels in place. They should not try to remove the label;
their exam can still be processed correctly.

**Listen carefully to all my instructions. I will give you time to complete each
step. Please look up after completing each step. Raise your hand if you have
any questions.**

Give students enough time to complete each step. Don’t move on until all students are ready.

Read the statements on the front cover of the Section I booklet. . . .

Sign your name and write today’s date. . . .

Now print your full legal name where indicated. . . .

Turn to the back cover of your exam booklet and read it completely. . . .

Are there any questions? . . .

You will now take the multiple-choice portion of the exam. You should have in
front of you the multiple-choice booklet and your answer sheet. You may never
discuss the multiple-choice exam content at any time in any form with anyone,
including your teacher and other students. If you disclose the multiple-choice
exam content through any means, your AP Exam score will be canceled.

Open your answer sheet to page 2. You must complete the answer sheet using
a No. 2 pencil only. Mark all of your responses beginning on page 2 of your
answer sheet, one response per question. Completely fill in the circles. If
you need to erase, do so carefully and completely. No credit will be given for
anything written in the exam booklet. Scratch paper is not allowed, but you may
use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 1 hour for Part A. Part A questions are numbered 1 through 30. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.



Note Start Time _____ . **Note Stop Time** _____ .

Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work.

After 50 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working on Part A and turn to page 24 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 90. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 45 minutes for Part B. You may begin.



Note Start Time _____ . **Note Stop Time** _____ .

Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other.

After 35 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working and turn to page 38. You have 3 minutes to answer Questions 91–94. These are survey questions and will not affect your score. Note that each survey question has five answer options. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions.

Then say:

Close your booklet and put your answer sheet on your desk, faceup. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label.

After all answer sheets have been collected, say:

Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, faceup. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II.

When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, notes, or textbooks during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you may never discuss the multiple-choice exam content with anyone, and if you disclose the content through any means, your AP Exam score will be canceled. Are there any questions? . . .



You may begin your break. Testing will resume at _____ .

SECTION II: Free Response

After the break, say:

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and, using your pen, complete Item 1 under "Important Identification Information." Print the first two letters of your last name and the first letter of your first name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

If this is your last AP Exam, you may keep your Student Pack. Place it under your chair for now. Otherwise I will collect all Student Packs. . . .

Read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs.

Then say:

Are there any questions? . . .

Section II also has two parts that are timed separately. You are responsible for pacing yourself and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions? . . .

You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra sheet of paper you use, write only your AP number and the question number you are working on. Do not write your name. Open your exam booklet and begin.



Note Start Time _____ . **Note Stop Time** _____ .

Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1's or 2's at the top of each page to assist you in monitoring their work.

After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .

Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may not use your calculator. Remember to show your work and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .

Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may go on to the next page and begin Part B.



Note Start Time _____ . **Note Stop Time** _____ .

After 50 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, faceup. . . .

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their exam booklets. Complete an Incident Report after the exam (see After-Exam Tasks below).

Then say:

Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed their initials and today’s date.
- Exam booklet back cover: The student completed the “Important Identification Information” area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

You may not discuss or share the free-response exam content with anyone unless it is released on the College Board website in about two days. Your AP Exam score results will be available online in July.

If you are giving the alternate exam for late testing, say:

None of the content in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

After-Exam Tasks

Be sure to give the completed seating chart to the AP coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

NOTE: If you administered exams to students with accommodations, review the *2017-18 AP Coordinator’s Manual* and the *2017-18 AP SSD Guidelines* for information about completing the NAR form, and returning these exams.

The exam proctor should complete the following tasks if asked to do so by the AP coordinator. Otherwise, the AP coordinator must complete these tasks:

- Complete an Incident Report for any students who used extra paper for the free-response section. (Incident Report forms are provided in the coordinator packets sent with the exam shipments.) **These forms must be completed with a No. 2 pencil.** It is best to complete a single Incident Report for multiple students per exam subject, per administration (regular or late testing), as long as all required information is provided. Include all exam booklets with extra sheets of paper in an Incident Report return envelope (see page 67 of the *2017-18 AP Coordinator’s Manual* for complete details).

- Return all exam materials to secure storage until they are shipped back to the AP Program. (See page 26 of the *2017-18 AP Coordinator's Manual* for more information about secure storage.) Before storing materials, check the “School Use Only” section on page 1 of the answer sheet and:
 - ◆ Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See “Post-Exam Activities” in the *2017-18 AP Coordinator's Manual*.
 - ◆ Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Student Answer Sheet for the Multiple-Choice Section

Use this section to capture student responses. (Note that the following answer sheet is a sample, and may differ from one used in an actual exam.)

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

- 76 (A) (B) (C) (D) (E)
- 77 (A) (B) (C) (D) (E)
- 78 (A) (B) (C) (D) (E)
- 79 (A) (B) (C) (D) (E)
- 80 (A) (B) (C) (D) (E)
- 81 (A) (B) (C) (D) (E)
- 82 (A) (B) (C) (D) (E)
- 83 (A) (B) (C) (D) (E)
- 84 (A) (B) (C) (D) (E)
- 85 (A) (B) (C) (D) (E)
- 86 (A) (B) (C) (D) (E)
- 87 (A) (B) (C) (D) (E)
- 88 (A) (B) (C) (D) (E)
- 89 (A) (B) (C) (D) (E)
- 90 (A) (B) (C) (D) (E)

- 91 (A) (B) (C) (D) (E)
- 92 (A) (B) (C) (D) (E)
- 93 (A) (B) (C) (D) (E)
- 94 (A) (B) (C) (D) (E)
- 95 (A) (B) (C) (D) (E)
- 96 (A) (B) (C) (D) (E)
- 97 (A) (B) (C) (D) (E)
- 98 (A) (B) (C) (D) (E)
- 99 (A) (B) (C) (D) (E)
- 100 (A) (B) (C) (D) (E)
- 101 (A) (B) (C) (D) (E)
- 102 (A) (B) (C) (D) (E)
- 103 (A) (B) (C) (D) (E)
- 104 (A) (B) (C) (D) (E)
- 105 (A) (B) (C) (D) (E)

- 106 (A) (B) (C) (D) (E)
- 107 (A) (B) (C) (D) (E)
- 108 (A) (B) (C) (D) (E)
- 109 (A) (B) (C) (D) (E)
- 110 (A) (B) (C) (D) (E)
- 111 (A) (B) (C) (D) (E)
- 112 (A) (B) (C) (D) (E)
- 113 (A) (B) (C) (D) (E)
- 114 (A) (B) (C) (D) (E)
- 115 (A) (B) (C) (D) (E)
- 116 (A) (B) (C) (D) (E)
- 117 (A) (B) (C) (D) (E)
- 118 (A) (B) (C) (D) (E)
- 119 (A) (B) (C) (D) (E)
- 120 (A) (B) (C) (D) (E)

QUESTIONS 121–126

For Students Taking AP Biology

Write your answer in the boxes at the top of the griddable area and fill in the corresponding circles. Mark only one circle in any column. You will receive credit only if the circles are filled in correctly.

121

		/	/	/	
-
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

122

		/	/	/	
-
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

123

		/	/	/	
-
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

124

		/	/	/	
-
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

125

		/	/	/	
-
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

126

		/	/	/	
-
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

QUESTIONS 131–142

For Students Taking AP Computer Science Principles, AP Physics 1, or AP Physics 2

Mark two responses per question. You will receive credit only if both correct responses are selected.

- 131 (A) (B) (C) (D)
- 132 (A) (B) (C) (D)
- 133 (A) (B) (C) (D)
- 134 (A) (B) (C) (D)

- 135 (A) (B) (C) (D)
- 136 (A) (B) (C) (D)
- 137 (A) (B) (C) (D)
- 138 (A) (B) (C) (D)

- 139 (A) (B) (C) (D)
- 140 (A) (B) (C) (D)
- 141 (A) (B) (C) (D)
- 142 (A) (B) (C) (D)



DO NOT WRITE IN THIS AREA

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2018 AP Exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP[®] Calculus BC Exam

SECTION I: Multiple Choice

2018

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour and 45 minutes

Number of Questions

45

Percent of Total Score

50%

Writing Instrument

Pencil required

Part A**Number of Questions**

30

Time

1 hour

Electronic Device

None allowed

Part B**Number of Questions**

15

Time

45 minutes

Electronic Device

Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 30 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 90 on page 3 of the answer sheet. Because Part A and Part B offer only four answer options for each question, do not mark the (E) answer circle for any question. The survey questions are numbers 91 through 94.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question Sample Answer

Chicago is a (A) ● (C) (D) (E)
(A) state
(B) city
(C) country
(D) continent

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4OBP4-S

68



1. If $y = x\sqrt{2x+5}$, then $y' =$

(A) $\frac{3x+5}{\sqrt{2x+5}}$

(B) $\frac{1}{\sqrt{2x+5}}$

(C) $\frac{1}{2\sqrt{2x+5}}$

(D) $\frac{5x+10}{2\sqrt{2x+5}}$

2. $\int 2^x dx =$

(A) $2^x + C$ (B) $(\ln 2)2^x + C$ (C) $\frac{2^x}{\ln 2} + C$ (D) $\frac{2^{x+1}}{x+1} + C$



3. $\lim_{x \rightarrow -7} \frac{x+7}{|x+7|}$ is

- (A) -1 (B) 0 (C) 1 (D) nonexistent

4. $\int \frac{(x^{1/3} - 4)^5}{6x^{2/3}} dx =$

(A) $\frac{(x^{1/3} - 4)^6}{12} + C$

(B) $\frac{(x^{1/3} - 4)^6}{6} + C$

(C) $\frac{5(x^{1/3} - 4)^4}{2} + C$

(D) $3(x^{1/3} - 4)^6 + C$

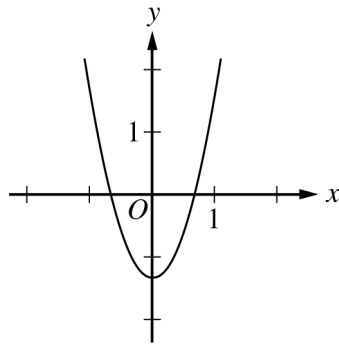


6. What is the sum of the series $\frac{\pi}{e} - \frac{\pi}{e^2} + \frac{\pi}{e^3} - \frac{\pi}{e^4} + \dots + (-1)^{n+1} \frac{\pi}{e^n} + \dots$?

- (A) $\frac{\pi}{e + \pi}$ (B) $\frac{\pi}{e + 1}$ (C) $\frac{\pi}{e - 1}$ (D) The series diverges.

7. If $f(x) = (x^2 + 1)^3$, what is $\lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1}$?

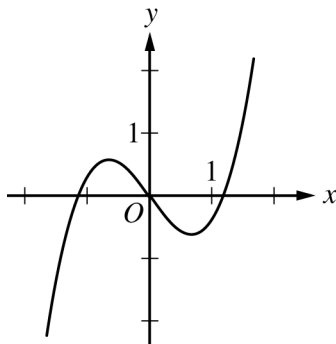
- (A) -24 (B) -8 (C) 0 (D) 12



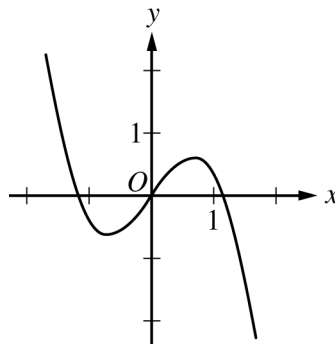
Graph of f''

8. The graph of f'' , the second derivative of the function f , is shown above. Which of the following could be the graph of f ?

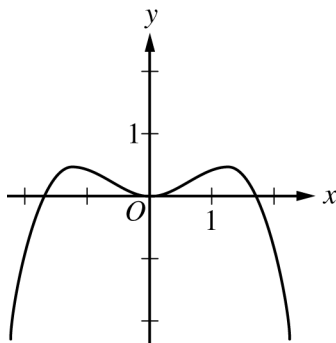
(A)



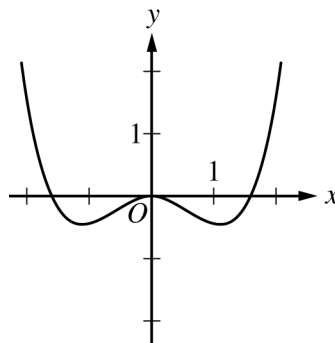
(B)



(C)



(D)



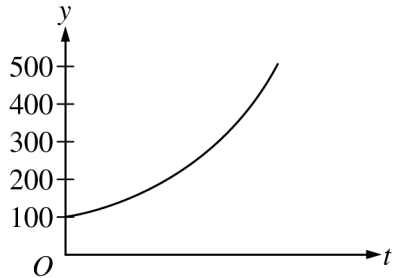


9. Let $y = f(x)$ be a differentiable function such that $\frac{dy}{dx} = \frac{x}{y}$ and $f(8) = 2$. What is the approximation of $f(8.1)$ using the line tangent to the graph of f at $x = 8$?
- (A) 0.4 (B) 2.025 (C) 2.4 (D) 6

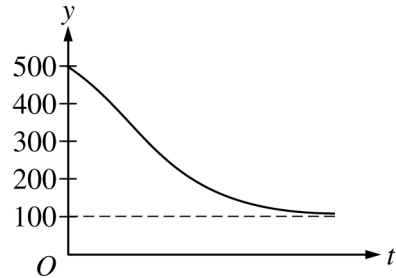
-
10. The series $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \dots + (-1)^n \frac{x^{2n}}{n!} + \dots$ converges to which of the following?
- (A) $\cos(x^2) + \sin(x^2)$ (B) $1 - x \sin x$ (C) $\cos x$ (D) e^{-x^2}

13. Which of the following graphs is the solution to the logistic differential equation $\frac{dy}{dt} = \frac{y}{5} \left(1 - \frac{y}{500} \right)$ with the initial condition $y(0) = 100$?

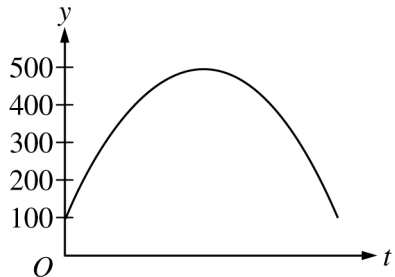
(A)



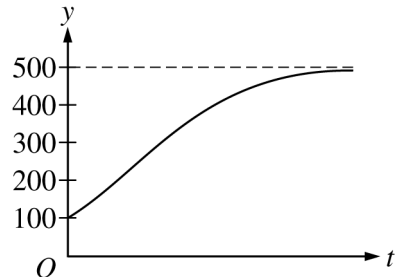
(B)



(C)

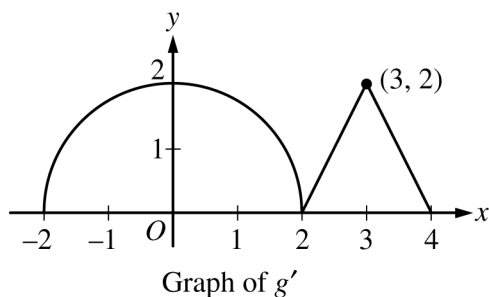


(D)



14. What is the absolute minimum value of $y = -\cos x - \sin x$ on the closed interval $\left[0, \frac{\pi}{2}\right]$?

- (A) $-2\sqrt{2}$ (B) -2 (C) $-\sqrt{2}$ (D) -1



15. The graph of g' , the first derivative of the function g , consists of a semicircle of radius 2 and two line segments, as shown in the figure above. If $g(0) = 1$, what is $g(3)$?

- (A) $\pi + 1$ (B) $\pi + 2$ (C) $2\pi + 1$ (D) $2\pi + 2$



20. Let g be a twice-differentiable, increasing function of t . If $g(0) = 20$ and $g(10) = 220$, which of the following must be true on the interval $0 < t < 10$?
- (A) $g'(t) = 0$ for some t in the interval.
 - (B) $g'(t) = 20$ for some t in the interval.
 - (C) $g''(t) = 0$ for some t in the interval.
 - (D) $g''(t) > 0$ for all t in the interval.

-
21. $\int \frac{6x^2 - 4x - 25}{x - 2} dx =$
- (A) $3x^2 + 8x - 9 \ln|x - 2| + C$
 - (B) $3x^2 + 8x + \frac{9}{(x - 2)^2} + C$
 - (C) $(2x^3 - 2x^2 - 25x) \ln|x - 2| + C$
 - (D) $\frac{2x^3 - 2x^2 - 25x}{\frac{x^2}{2} - 2x} + C$



22. Which of the following statements about convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ is true?

- (A) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.
- (B) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
- (C) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.
- (D) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.



23. The length of the curve $y = \sin x$ from $x = 0$ to $x = \frac{3\pi}{4}$ is given by

(A) $\int_0^{3\pi/4} \sin x \, dx$

(B) $\int_0^{3\pi/4} \sqrt{1 + \sin^2 x} \, dx$

(C) $\int_0^{3\pi/4} \sqrt{1 - \cos^2 x} \, dx$

(D) $\int_0^{3\pi/4} \sqrt{1 + \cos^2 x} \, dx$



24. Let f be a function such that $f'(x) = \sin(x^2)$ and $f(0) = 0$. What are the first three nonzero terms of the Maclaurin series for f ?

(A) $x - \frac{x^5}{10} + \frac{x^9}{216}$

(B) $2x - x^5 + \frac{x^9}{12}$

(C) $\frac{x^3}{3} - \frac{x^7}{21} + \frac{x^{11}}{55}$

(D) $\frac{x^3}{3} - \frac{x^7}{42} + \frac{x^{11}}{1320}$



28. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{3p+1}} dx$ converges?

- (A) $p < 0$ (B) $p > -\frac{1}{3}$ (C) $p > 0$ (D) $p > 1$

29. The temperature of a solid at time $t \geq 0$ is modeled by the nonconstant function H and increases according to the differential equation $\frac{dH}{dt} = 2H + 1$, where $H(t)$ is measured in degrees Fahrenheit and t is measured in hours. Which of the following must be true?

- (A) $H = H^2 + t + C$
(B) $\ln|2H + 1| = \frac{t}{2} + C$
(C) $\ln|2H + 1| = t + C$
(D) $\ln|2H + 1| = 2t + C$

PART B STARTS ON PAGE 26.

B**B****B****B****B****B****B****B****B****CALCULUS BC****SECTION I, Part B****Time—45 minutes****Number of questions—15****A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.**

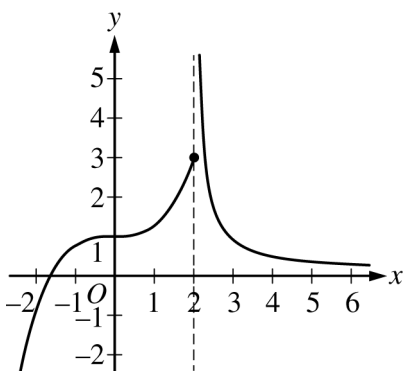
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

B**B****B****B****B****B****B****B****B**Graph of f

76. The graph of the function f is shown in the figure above. Which of the following statements must be false?

- (A) $\lim_{x \rightarrow 2^-} f(x) = 3$
- (B) $\lim_{x \rightarrow 2^+} f(x) = \infty$
- (C) $\lim_{x \rightarrow 2} f(x) = f(2)$
- (D) $\lim_{x \rightarrow \infty} f(x) = 0$

B**B****B****B****B****B****B****B****B**

77. The rate at which water leaks from a tank, in gallons per hour, is modeled by R , a differentiable function of the number of hours after the leak is discovered. Which of the following is the best interpretation of $R'(3)$?
- (A) The amount of water, in gallons, that has leaked out of the tank during the first three hours after the leak is discovered
 - (B) The amount of change, in gallons per hour, in the rate at which water is leaking during the three hours after the leak is discovered
 - (C) The rate at which water leaks from the tank, in gallons per hour, three hours after the leak is discovered
 - (D) The rate of change of the rate at which water leaks from the tank, in gallons per hour per hour, three hours after the leak is discovered

-
78. On a certain day, the temperature, in degrees Fahrenheit, in a small town t hours after midnight ($t = 0$) is modeled by the function $g(t) = 65 - 8 \sin\left(\frac{\pi t}{12}\right)$. What is the average temperature in the town between 3 A.M. ($t = 3$) and 6 A.M. ($t = 6$), in degrees Fahrenheit?
- (A) 57.609 (B) 57.797 (C) 58.172 (D) 59.907

B**B****B****B****B****B****B****B****B**

x	0.0	0.5	1.0	1.5	2.0
$f'(x)$	1.0	0.7	0.5	0.4	0.3

79. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = f'(x)$ with initial condition $f(1) = 5$. Selected values of $f'(x)$ are given in the table above. What is the approximation for $f(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

- (A) 5.35 (B) 5.45 (C) 5.50 (D) 5.90

B**B****B****B****B****B****B****B****B**

80. The first derivative of the function f is defined by $f'(x) = (x^2 + 1)\sin(3x - 1)$ for $-1.5 < x < 1.5$. On which of the following intervals is the graph of f concave up?
- (A) $(-1.5, -1.341)$ and $(-0.240, 0.964)$
(B) $(-1.341, -0.240)$ and $(0.964, 1.5)$
(C) $(-0.714, 0.333)$ and $(1.381, 1.5)$
(D) $(-1.5, -0.714)$ and $(0.333, 1.381)$
-

81. For $t \geq 0$, the velocity of a particle moving along the x -axis is given by $v(t) = t^3 - 6t^2 + 10t - 4$. At what time t does the direction of motion of the particle change from right to left?
- (A) 0.586 (B) 1.184 (C) 2.000 (D) 2.816

B**B****B****B****B****B****B****B****B**

82. Let f be a function such that $f(1) = -2$ and $f(5) = 7$. Which of the following conditions ensures that $f(c) = 0$ for some value c in the open interval $(1, 5)$?

(A) $\int_1^5 f(x) dx$ exists.

(B) f is increasing on the closed interval $[1, 5]$.

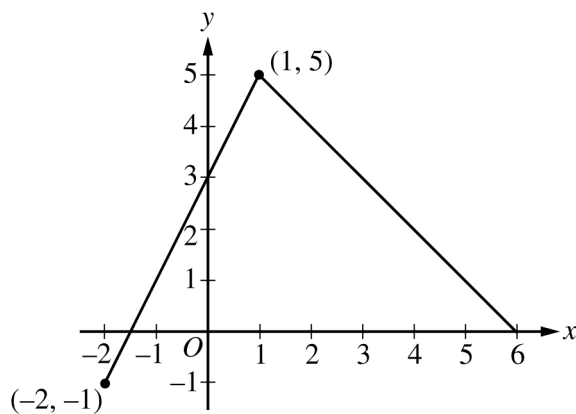
(C) f is continuous on the closed interval $[1, 5]$.

(D) f is defined for all values of x in the closed interval $[1, 5]$.

83. For time $t > 0$, the position of an object moving in the xy -plane is given by the parametric equations

$x(t) = t \cos\left(\frac{t}{2}\right)$ and $y(t) = \sqrt{t^2 + 2t}$. What is the speed of the object at time $t = 1$?

(A) 1.155 (B) 1.319 (C) 1.339 (D) 1.810

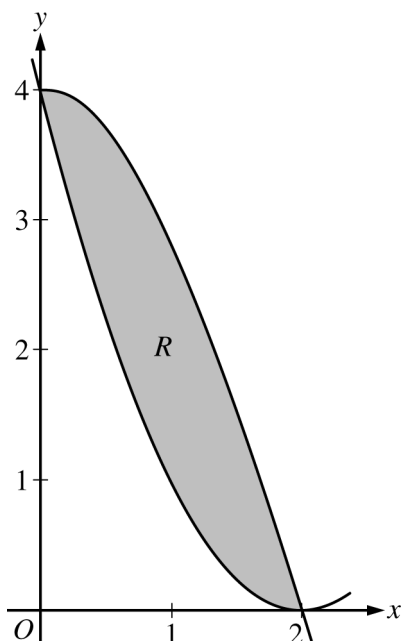
B**B****B****B****B****B****B****B****B**Graph of g

84. The graph of the function g is shown above. If f is the function given by $f(x) = g(g(x))$, what is the value of $f'(0)$?

- (A) -2 (B) -1 (C) 2 (D) 3

85. Let f be a function such that $f(-x) = -f(x)$ for all x . If $\int_0^2 f(x) dx = 5$, then $\int_{-2}^2 (f(x) + 6) dx =$

- (A) 6 (B) 16 (C) 24 (D) 34

B**B****B****B****B****B****B****B****B**

86. Let R be the region in the first quadrant bounded by the graphs of $y = 4 \cos\left(\frac{\pi x}{4}\right)$ and $y = (x - 2)^2$, as shown in the figure above. The region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in region R . What is the volume of the solid?

(A) 1.775 (B) 3.549 (C) 4.800 (D) 5.575

B**B****B****B****B****B****B****B****B**

87. Suppose $\lim_{n \rightarrow \infty} a_n = \infty$ and $a_{n+1} \geq a_n > 0$ for all $n \geq 1$. Which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.

(B) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(C) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$ converges.

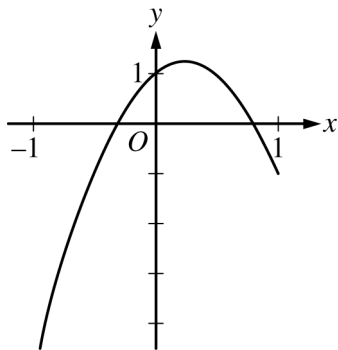
88. At time $t \geq 0$, a particle moving in the xy -plane has velocity vector given by $v(t) = \langle 3, 2^{-t^2} \rangle$. If the particle is at the point $\left(1, \frac{1}{2}\right)$ at time $t = 0$, how far is the particle from the origin at time $t = 1$?

- (A) 2.304 (B) 3.107 (C) 4.209 (D) 5.310

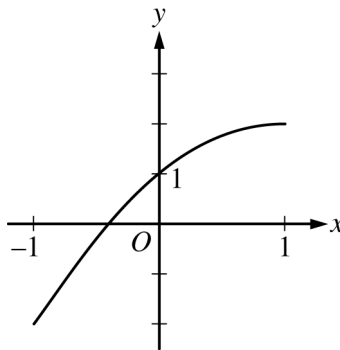
B**B****B****B****B****B****B****B****B**

89. Let f be a function with $f(0) = 1$, $f'(0) = 2$, and $f''(0) = -2$. Which of the following could be the graph of the second-degree Taylor polynomial for f about $x = 0$?

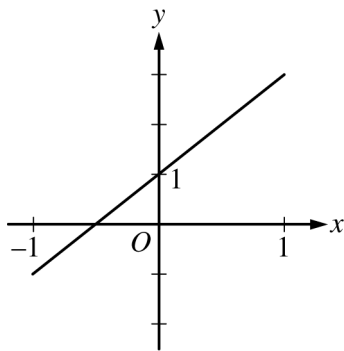
(A)



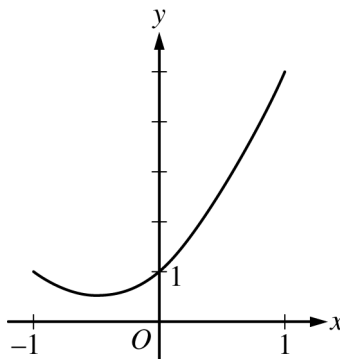
(B)



(C)



(D)



B**B****B****B****B****B****B****B****B**

90. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges to S . Based on the alternating series error bound, what is the least number of terms in the series that must be summed to guarantee a partial sum that is within 0.03 of S ?

(A) 34 (B) 333 (C) 1111 (D) 9999

B

B

B

B

B

B

B

B

B

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

**AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND
ANSWER QUESTIONS 91–94.**

Section II: Free-Response Questions

This is the free-response section of the 2018 AP Exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP[®] Calculus BC Exam

SECTION II: Free Response

2018

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour and 30 minutes

Number of Questions

6

Percent of Total Score

50%

Writing Instrument

Either pencil or pen with black or dark blue ink

Weight

The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions

2

Time

30 minutes

Electronic Device

Graphing calculator required

Percent of Section II Score

33.33%

Part B

Number of Questions

4

Time

1 hour

Electronic Device

None allowed

Percent of Section II Score

66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name
First letter of your first name
2. Date of birth

Month Day Year
3. Six-digit school code
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
No, I do not grant the College Board these rights.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as `fnInt(X2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I

Form Code 40BP4-S

68

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g , where $g(t)$ is measured in cubic feet per minute for $0 \leq t \leq 8$ minutes. Selected values of $g(t)$ are given in the table above.
- (a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

-
- (b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

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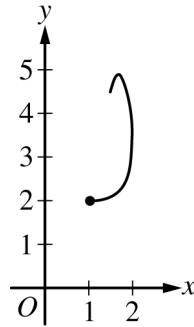
1**1****1****1****1****1****1****1****1****1**

- (c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \leq t \leq 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

-
- (d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

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2. A particle moving in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$, where $\frac{dx}{dt} = \cos(t^2)$ and $\frac{dy}{dt} = e^t \sin(t^2)$. At time $t = 0$, the particle is at position $(1, 2)$. The figure above shows the path of the particle for $0 \leq t \leq 2$.

(a) Find the position of the particle at time $t = 2$.

(b) Find the slope of the line tangent to the particle's path at time $t = 2$.

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2**2****2****2****2****2****2****2****2****2**

(c) Find the speed of the particle at time $t = 2$. Find the acceleration vector of the particle at time $t = 2$.

(d) Consider a rectangle with vertices at points $(0, 0)$, $(x(t), 0)$, $(x(t), y(t))$, and $(0, y(t))$ at time $t \geq 0$. For $0 \leq t \leq 2$, at what time t is the perimeter of the rectangle a maximum? Justify your answer.

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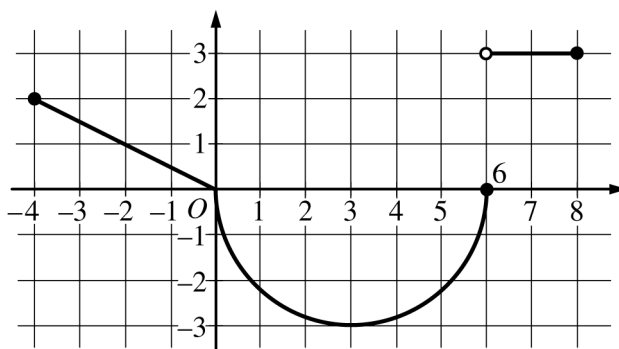
END OF PART A
IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—1 hour
Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t) dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

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NO CALCULATOR ALLOWED

(c) For each of $\lim_{x \rightarrow 0^-} g'(x)$ and $\lim_{x \rightarrow 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim_{x \rightarrow -2} \frac{f(x) + 7}{e^{3x+6} - 1}$.

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NO CALCULATOR ALLOWED

4. Let g be the function that satisfies $g(0) = 0$ and whose derivative satisfies $g'(x) = 2|x|$.

(a) Find expressions for $g(x)$ and $g''(x)$.

(b) Find the x -coordinate, if any, of each point of inflection of the graph of $y = g(x)$. Explain your reasoning.

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NO CALCULATOR ALLOWED

(c) Let $h(x) = \int_0^x \sqrt{1 + 4t^2} dt$. For $x \geq 0$, $h(x)$ is the length of the graph of g from $t = 0$ to $t = x$. Use

Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(4)$.

(d) Find the value of $\int_{\pi/2}^{\pi} g'(x) \cos x dx$.

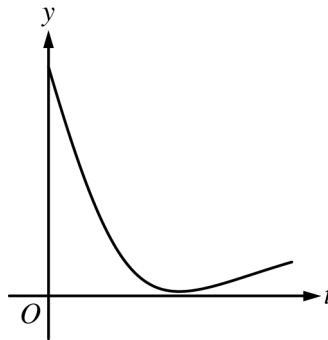
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NO CALCULATOR ALLOWED

5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction ($t = 0$), there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.
- (a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

-
- (b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.



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NO CALCULATOR ALLOWED

- (c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

-
- (d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

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NO CALCULATOR ALLOWED

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$
0	4	5	-1	$-\frac{15}{2}$	23
1	8	3	-2	$\frac{3}{2}$	$\frac{2}{5}$

6. Let f be a function having derivatives of all orders for all real numbers. Selected values of f and its first four derivatives are shown in the table above.

(a) Write the second-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

(b) Let g be a function such that $g(x) = f(x^3)$. Write the fifth-degree Taylor polynomial for g' , the derivative of g , about $x = 0$.

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NO CALCULATOR ALLOWED

(c) Write the third-degree Taylor polynomial for f about $x = 1$.

(d) It is known that $|f^{(4)}(x)| \leq 300$ for $0 \leq x \leq 1.125$. The third-degree Taylor polynomial for f about $x = 1$, found in part (c), is used to approximate $f(1.1)$. Use the Lagrange error bound along with the information about $f^{(4)}(x)$ to find an upper bound on the error of the approximation.

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STOP
END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to the multiple-choice questions in this exam.

**Answer Key for AP Calculus BC
Practice Exam, Section I**

Question 1: A	Question 24: D
Question 2: C	Question 25: C
Question 3: D	Question 26: D
Question 4: A	Question 27: A
Question 5: B	Question 28: C
Question 6: B	Question 29: D
Question 7: A	Question 30: B
Question 8: D	Question 76: C
Question 9: C	Question 77: D
Question 10: D	Question 78: B
Question 11: D	Question 79: B
Question 12: B	Question 80: A
Question 13: D	Question 81: C
Question 14: C	Question 82: C
Question 15: B	Question 83: B
Question 16: A	Question 84: A
Question 17: A	Question 85: C
Question 18: D	Question 86: A
Question 19: B	Question 87: D
Question 20: B	Question 88: C
Question 21: A	Question 89: B
Question 22: C	Question 90: C
Question 23: D	

Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

AP[®] CALCULUS BC
2018 SCORING GUIDELINES

Question 1

(a) $g'(3) \approx \frac{g(5) - g(1)}{5 - 1} = \frac{20.5 - 15.1}{4} = 1.35$

At time $t = 3$ minutes, the rate at which grain is being added to the silo is increasing at a rate of 1.35 cubic feet per minute per minute.

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{interpretation with units} \end{cases}$

(b) The total amount of grain added to the silo from time $t = 0$ to time $t = 8$ is $\int_0^8 g(t) dt$ cubic feet.

$$\int_0^8 g(t) dt \approx g(1) \cdot (1 - 0) + g(5) \cdot (5 - 1) + g(6) \cdot (6 - 5) + g(8) \cdot (8 - 6)$$

$$= 15.1 \cdot 1 + 20.5 \cdot 4 + 18.3 \cdot 1 + 22.7 \cdot 2 = 160.8$$

3 : $\begin{cases} 1 : \text{integral expression} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) $\int_0^8 w(t) dt = 99.051497$

The approximate amount of unspoiled grain remaining in the silo at time $t = 8$ is $160.8 - \int_0^8 w(t) dt = 61.749$ (or 61.748) cubic feet.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $g(6) - w(6) = 18.3 - 16.063173 = 2.236827 > 0$

Because $g(6) - w(6) > 0$, the amount of unspoiled grain is increasing at time $t = 6$.

2 : $\begin{cases} 1 : \text{considers } g(6) - w(6) \\ 1 : \text{answer} \end{cases}$

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2018 SCORING GUIDELINES

Question 2

(a) $x(2) = 1 + \int_0^2 \frac{dx}{dt} dt = 1.461461$

$$y(2) = 2 + \int_0^2 \frac{dy}{dt} dt = 4.268236$$

The position of the particle at time $t = 2$ is $(1.461, 4.268)$.

(b) $\left. \frac{dy}{dx} \right|_{t=2} = \left. \frac{dy/dt}{dx/dt} \right|_{t=2} = 8.555206$

The slope of the line tangent to the particle's path at time $t = 2$ is 8.555.

(c) $\left. \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \right|_{t=2} = 5.630128$

The speed of the particle at time $t = 2$ is 5.630.

$$x''(2) = 3.027210$$

$$y''(2) = -24.911294$$

The acceleration vector of the particle at time $t = 2$ is $\langle 3.027, -24.911 \rangle$.

(d) Perimeter = $p(t) = 2(x(t) + y(t))$

$$p'(t) = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 0 \Rightarrow t = 1.721837$$

Because $t = 1.721837$ is the only interior critical point for $0 \leq t \leq 2$ and $p'(t)$ changes from positive to negative at this point, the perimeter of the rectangle is a maximum at $t = 1.722$ (or $t = 1.721$).

3 : $\left\{ \begin{array}{l} 1 : \text{integrals} \\ 1 : \text{uses initial condition} \\ 1 : \text{position} \end{array} \right.$

1 : slope

2 : $\left\{ \begin{array}{l} 1 : \text{speed} \\ 1 : \text{acceleration vector} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 1 : \text{sets } p'(t) = 0 \\ 2 : \text{answer with justification} \end{array} \right.$

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2018 SCORING GUIDELINES

Question 3

(a) $f(7) = 3 \cdot 7 + \int_0^7 g(t) dt = 21 - \frac{9\pi}{2} + 3 = 24 - \frac{9\pi}{2}$
 $f'(7) = 3 + g(7) = 3 + 3 = 6$

$$2 : \begin{cases} 1 : f(7) \\ 1 : f'(7) \end{cases}$$

(b) On the interval $-4 \leq x \leq 3$, $f'(x) = 3 + g(x)$.
Because $f'(x) \geq 0$ for $-4 \leq x \leq 3$, f is nondecreasing over the entire interval, and the maximum must occur when $x = 3$.

2 : answer with justification

(c) $\lim_{x \rightarrow 0^-} g'(x) = -\frac{1}{2}$
 $\lim_{x \rightarrow 0^+} g'(x)$ does not exist.

$$2 : \begin{cases} 1 : \text{left-hand limit} \\ 1 : \text{right-hand limit} \end{cases}$$

(d) $\lim_{x \rightarrow -2} (f(x) + 7) = -6 + \int_0^{-2} g(t) dt + 7 = 0$
 $\lim_{x \rightarrow -2} (e^{3x+6} - 1) = 0$

$$3 : \begin{cases} 1 : \text{limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow -2} \frac{f(x) + 7}{e^{3x+6} - 1} = \lim_{x \rightarrow -2} \frac{f'(x)}{3e^{3x+6}} = \frac{3 + g(-2)}{3} = \frac{3 + 1}{3} = \frac{4}{3}.$$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

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Question 4

(a)
$$g(x) = \begin{cases} -x^2 & \text{for } x < 0 \\ 0 & \text{for } x = 0 \\ x^2 & \text{for } x > 0 \end{cases}$$

$$g''(x) = \begin{cases} -2 & \text{for } x < 0 \\ 2 & \text{for } x > 0 \end{cases}$$

$$2 : \begin{cases} 1 : g(x) \\ 1 : g''(x) \end{cases}$$

(b) The only point of inflection of the graph of $y = g(x)$ occurs at $x = 0$, where $g''(x)$ changes sign from negative to positive.

1 : answer with explanation

(c)
$$h(x) = \int_0^x \sqrt{1+4t^2} dt \Rightarrow h'(x) = \sqrt{1+4x^2}$$

$$h(0) = 0$$

$$h(2) \approx h(0) + 2 \cdot h'(0) = 0 + 2 \cdot 1 = 2$$

$$h(4) \approx h(2) + 2 \cdot h'(2) \approx 2 + 2\sqrt{17}$$

$$3 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{Euler's method with two steps} \\ 1 : \text{approximation} \end{cases}$$

(d) $g'(x) = 2x$ for $x > 0$

$$u = 2x \quad dv = \cos x \, dx$$

$$du = 2 \, dx \quad v = \sin x$$

$$\begin{aligned} \int_{\pi/2}^{\pi} 2x \cos x \, dx &= 2x \sin x \Big|_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} 2 \sin x \, dx \\ &= 2x \sin x \Big|_{\pi/2}^{\pi} + 2 \cos x \Big|_{\pi/2}^{\pi} \\ &= 0 - \pi + (-2) - 0 \\ &= -\pi - 2 \end{aligned}$$

$$3 : \begin{cases} 2 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$$

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2018 SCORING GUIDELINES

Question 5

(a) $y'(0) = -0.02(10^2) = -2$

An equation for the line tangent to the graph of $y = f(t)$ at $t = 0$ is $y = 10 - 2t$.

$$y(2) \approx 10 - 2(2) = 6 \text{ grams}$$

(b) $\frac{dy}{dt} = -0.02y^2 \leq 0$, so the graph of f is nonincreasing.

The graph of f cannot resemble the given graph because the given graph is increasing on a portion of its domain.

(c) $\int \left(-\frac{1}{y^2} \right) dy = \int 0.02 dt$

$$\frac{1}{y} = 0.02t + C$$

$$\frac{1}{10} = 0.02(0) + C \Rightarrow C = 0.1$$

$$\frac{1}{y} = 0.02t + 0.1 \Rightarrow y = \frac{1}{0.02t + 0.1} = \frac{50}{t + 5}$$

Note: this solution is valid for $t > -5$.

(d) $\frac{d^2y}{dt^2} = -0.04y \frac{dy}{dt}$
 $= -0.04y(-0.02y^2)$
 $= 0.0008y^3$

Because $y > 0$, $0.0008y^3 > 0$.

The amount of substance is changing at an increasing rate.

— OR —

From part (c), $f(t) = \frac{50}{t + 5}$, and from context, $t \geq 0$.

$$f'(t) = \frac{-50}{(t + 5)^2} \text{ and } f''(t) = \frac{100}{(t + 5)^3} > 0 \text{ for } t \geq 0.$$

The amount of substance is changing at an increasing rate.

$$2 : \begin{cases} 1 : y'(0) \\ 1 : \text{approximation} \end{cases}$$

1 : answer with reason

$$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{answer} \end{cases}$$

Note: max 2/4 [1-1-0-0] if no constant of integration

Note: 0/4 if no separation of variables

$$2 : \begin{cases} 1 : \frac{d^2y}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$$

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2018 SCORING GUIDELINES

Question 6

- (a) The second-degree Taylor polynomial for f about $x = 0$ is $4 + 5x - \frac{1}{2}x^2$.

$$f(0.2) \approx 4 + 5(0.2) - \frac{1}{2}(0.04) = 4.98$$

3 : $\left\{ \begin{array}{l} 2 : \text{second-degree} \\ \text{Taylor polynomial} \\ 1 : \text{approximation} \end{array} \right.$

- (b) The fifth-degree Taylor polynomial for g' about $x = 0$ is

$$\frac{d}{dx} \left(4 + 5(x^3) - \frac{1}{2}(x^3)^2 \right) = 15x^2 - 3x^5.$$

2 : $\left\{ \begin{array}{l} 1 : \text{substitution} \\ 1 : \text{answer} \end{array} \right.$

- (c) The third-degree Taylor polynomial for f about $x = 1$ is

$$\begin{aligned} 8 + 3(x-1) - \frac{2}{2}(x-1)^2 + \frac{3/2}{3!}(x-1)^3 \\ = 8 + 3(x-1) - (x-1)^2 + \frac{1}{4}(x-1)^3. \end{aligned}$$

2 : $\left\{ \begin{array}{l} 1 : \text{two terms} \\ 1 : \text{remaining terms} \end{array} \right.$

- (d) $\frac{\max_{1 \leq x \leq 1.1} |f^{(4)}(x)|}{4!} \cdot (1.1-1)^4 \leq \frac{300}{4!} \cdot (1.1-1)^4 = \frac{300}{24} \cdot 0.1^4 = \frac{1}{800}$

2 : $\left\{ \begin{array}{l} 1 : \text{form of the error bound} \\ 1 : \text{answer} \end{array} \right.$

An upper bound on the error of the approximation is $\frac{1}{800}$.

Scoring Worksheets

The following provides scoring worksheets and conversion tables used for calculating a composite score of the exam.

2018 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

$$\frac{\text{Number Correct}}{\text{(out of 45)}} \times 1.2000 = \frac{\text{Weighted Section I Score}}{\text{(Do not round)}}$$

Section II: Free Response

$$\text{Question 1 } \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 2 } \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 3 } \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 4 } \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 5 } \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 6 } \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Sum} = \frac{\text{_____}}{\text{Weighted Section II Score (Do not round)}}$$

Composite Score

$$\frac{\text{Weighted Section I Score}}{\text{_____}} + \frac{\text{Weighted Section II Score}}{\text{_____}} = \frac{\text{Composite Score (Round to nearest whole number)}}{\text{_____}}$$

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
66-108	5
55-65	4
39-54	3
23-38	2
0-22	1

2018 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (1-5, 7-9, 12, 14-15, 17-18, 20-21, 26-27, 29, 76-78, 80-82, 84-86)

$$\frac{\text{Number Correct}}{\text{(out of 27)}} \times 1.0000 = \frac{\text{Weighted Section I Score}}{\text{(Do not round)}}$$

Section II: Free Response

$$\text{Question 1} \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 3} \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 5} \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Sum} = \frac{\text{_____}}{\text{Weighted Section II Score (Do not round)}}$$

Composite Score

$$\frac{\text{Weighted Section I Score}}{\text{_____}} + \frac{\text{Weighted Section II Score}}{\text{_____}} = \frac{\text{Composite Score (Round to nearest whole number)}}{\text{_____}}$$

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Score
34-54	5
27-33	4
20-26	3
13-19	2
0-12	1

Question Descriptors and Performance Data

The following contains tables showing the content assessed, the correct answer, and how AP students performed on each question.

2018 AP Calculus BC Question Descriptors and Performance Data

Multiple-Choice Questions

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus 1	Mathematical Practice for AP Calculus 2	Key	% Correct
1	2.1C	2.1C3	Implementing algebraic/computational processes	Building notational fluency	A	67
2	3.3B(a)	3.3B3	Implementing algebraic/computational processes	Building notational fluency	C	58
3	1.1A(b)	1.1A3	Reasoning with definitions and theorems	Building notational fluency	D	35
4	3.3B(a)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	A	70
5	3.2B	3.2B2	Implementing algebraic/computational processes	Connecting multiple representations	B	89
6	4.1B	4.1B1	Implementing algebraic/computational processes	Building notational fluency	B	43
7	2.1A	2.1A2	Reasoning with definitions and theorems	Implementing algebraic/computational processes	A	62
8	2.2A	2.2A3	Connecting concepts	Connecting multiple representations	D	72
9	2.3B	2.3B2	Connecting concepts	Implementing algebraic/computational processes	C	83
10	4.2B	4.2B2	Implementing algebraic/computational processes	Building notational fluency	D	51
11	3.3B(a)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	D	65
12	2.1C	2.1C5	Implementing algebraic/computational processes	Connecting concepts	B	69
13	3.5A	3.5A1	Connecting multiple representations	Connecting concepts	D	86
14	2.2A	2.2A1	Implementing algebraic/computational processes	Connecting concepts	C	64
15	3.2C	3.2C1	Connecting multiple representations	Implementing algebraic/computational processes	B	66
16	3.3B(a)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	A	31
17	2.3C	2.3C2	Implementing algebraic/computational processes	Connecting concepts	A	80
18	3.3A	3.3A2	Connecting concepts	Implementing algebraic/computational processes	D	59
19	2.1C	2.1C7	Implementing algebraic/computational processes	Connecting concepts	B	46
20	2.4A	2.4A1	Reasoning with definitions and theorems	Connecting concepts	B	67
21	3.3B(a)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	A	51
22	4.1A	4.1A6	Reasoning with definitions and theorems	Implementing algebraic/computational processes	C	64
23	3.4D	3.4D3	Connecting concepts	Reasoning with definitions and theorems	D	81
24	4.2B	4.2B5	Implementing algebraic/computational processes	Building notational fluency	D	40
25	3.4D	3.4D1	Connecting multiple representations	Reasoning with definitions and theorems	C	55
26	2.2A	2.2A1	Connecting multiple representations	Reasoning with definitions and theorems	D	36
27	2.3F	2.3F1	Connecting multiple representations	Connecting concepts	A	73
28	3.2D	3.2D2	Connecting concepts	Reasoning with definitions and theorems	C	68
29	3.5A	3.5A2	Connecting concepts	Implementing algebraic/computational processes	D	41
30	4.2C	4.2C1	Reasoning with definitions and theorems	Connecting concepts	B	17

2018 AP Calculus BC Question Descriptors and Performance Data

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus 1	Mathematical Practice for AP Calculus 2	Key	% Correct
76	1.2A	1.2A1	Connecting multiple representations	Reasoning with definitions and theorems	C	91
77	2.3A	2.3A2	Connecting concepts	Building notational fluency	D	81
78	3.4B	3.4B1	Implementing algebraic/computational processes	Connecting concepts	B	85
79	2.3F	2.3F2	Implementing algebraic/computational processes	Connecting multiple representations	B	83
80	2.2A	2.2A1	Connecting concepts	Implementing algebraic/computational processes	A	71
81	2.3C	2.3C1	Implementing algebraic/computational processes	Connecting concepts	C	76
82	1.2B	1.2B1	Reasoning with definitions and theorems	Building notational fluency	C	83
83	2.3C	2.3C4	Reasoning with definitions and theorems	Implementing algebraic/computational processes	B	71
84	2.1C	2.1C4	Connecting multiple representations	Implementing algebraic/computational processes	A	71
85	3.2C	3.2C2	Reasoning with definitions and theorems	Implementing algebraic/computational processes	C	46
86	3.4D	3.4D2	Implementing algebraic/computational processes	Connecting multiple representations	A	54
87	4.1A	4.1A6	Reasoning with definitions and theorems	Building notational fluency	D	52
88	3.4C	3.4C2	Implementing algebraic/computational processes	Connecting concepts	C	61
89	4.2A	4.2A1	Connecting multiple representations	Connecting concepts	B	52
90	4.1B	4.1B2	Reasoning with definitions and theorems	Implementing algebraic/computational processes	C	62

2018 AP Calculus BC Question Descriptors and Performance Data

Free-Response Questions

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus	Mean
1	2.1B 2.3A 2.3A 3.2B 3.3B(b) 3.4A	2.1B1 2.3A1 2.3A2 3.2B2 3.3B2 3.4A2	Reasoning with definitions and theorems Connecting concepts Implementing algebraic/computational processes Connecting multiple representations Building notational fluency Communicating	6.23
2	1.2B 2.1C 2.3C 2.3C 3.4C	1.2B1 2.1C7 2.3C3 2.3C4 3.4C2	Reasoning with definitions and theorems Connecting concepts Implementing algebraic/computational processes Connecting multiple representations Building notational fluency Communicating	4.67
3	1.1A(b) 1.1B 1.1C 2.2A 3.2C 3.2C 3.3A	1.1A3 1.1B1 1.1C3 2.2A1 3.2C1 3.2C3 3.3A2	Reasoning with definitions and theorems Connecting concepts Implementing algebraic/computational processes Connecting multiple representations Building notational fluency Communicating	4.19
4	2.1C 2.2A 2.2B 2.3F 3.3A 3.3B(b)	2.1C2 2.2A1 2.2B1 2.3F2 3.3A2 3.3B5	Reasoning with definitions and theorems Connecting concepts Implementing algebraic/computational processes Building notational fluency Communicating	3.22
5	2.1C 2.1D 2.2A 2.3B 3.5A	2.1C4 2.1D1 2.2A1 2.3B2 3.5A2	Reasoning with definitions and theorems Connecting concepts Implementing algebraic/computational processes Connecting multiple representations Building notational fluency Communicating	3.95
6	4.2A 4.2A 4.2A 4.2B	4.2A1 4.2A2 4.2A4 4.2B5	Reasoning with definitions and theorems Connecting concepts Implementing algebraic/computational processes Connecting multiple representations Building notational fluency Communicating	4.00

AP Calculus BC

The College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT[®] and the Advanced Placement Program[®]. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.